Improving Learning in Mathematics

Malcolm Swan

Shell/ MARS

University of Nottingham

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Outline

- Context and need for a new approach to the learning of concepts
- Research background
- The power of lesson ‘Genres’
- Impact on teachers and students.

Context and need for a new approach

‘At-risk’ students aged 16-19

- In England, the major end qualification for the compulsory phase of education is the GCSE.
- Each year, 16-19 year-old students that fail to attain the minimum grade required for planned careers or entry into higher education embark on re-sit courses within further education (FE) institutions.
- National inspection reports reveal that the teaching on these courses is teacher-centred and transmission-oriented.

‘At-risk’ students aged 16-19

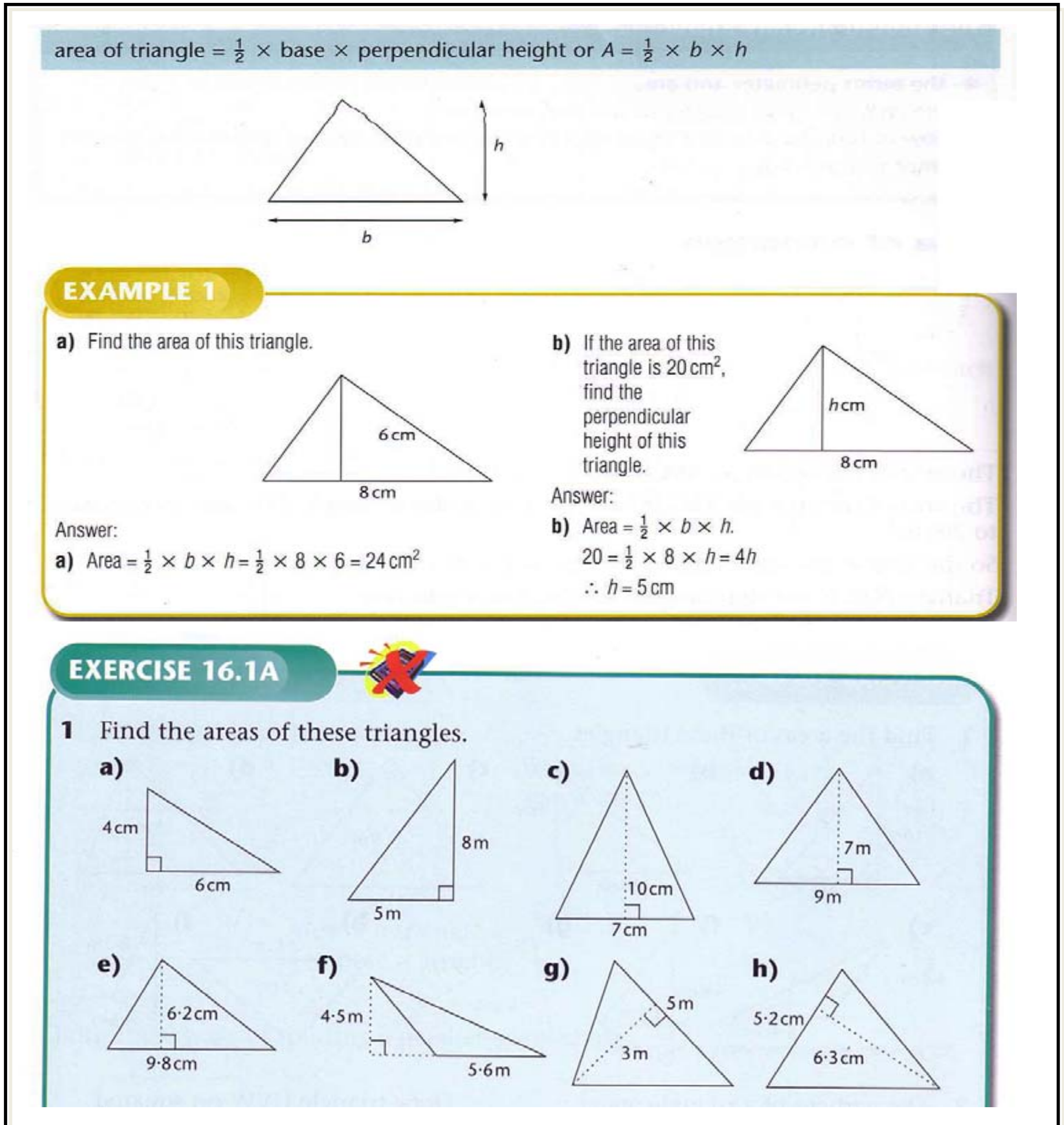
- 55% fail to attain GCSE grade C at 16.
- Colleges over-recruit.
- High drop out rates.
- Average level of attendance is only 63%.
- Teaching is ‘narrow and unimaginative’.
- Pass rates poor.

(Further Education Funding Council (1999)).

‘Transmission’ culture

- **Mathematics** is seen as
  - a body of knowledge and procedures to be ‘covered’
- **Learning** is seen as:
  - an individual activity based on listening and imitating
- **Teaching** is seen as
  - structuring a linear curriculum for the students
  - giving explanations and checking these have been understood through practice questions
  - ‘correcting’ misunderstandings when students fail to ‘grasp’ what is taught

Textbook dominated



area of triangle = $\frac{1}{2}$ × base × perpendicular height or $A = \frac{1}{2} \times b \times h$

EXAMPLE 1

a) Find the area of this triangle.

Answer:

a) Area = $\frac{1}{2} \times b \times h = \frac{1}{2} \times 8 \times 6 = 24\,\text{cm}^2$

b) If the area of this triangle is 20 cm², find the perpendicular height of this triangle.

Answer:

b) Area = $\frac{1}{2} \times b \times h$.

$20 = \frac{1}{2} \times 8 \times h = 4h$

$\therefore h = 5\,\text{cm}$

EXERCISE 16.1A

1 Find the areas of these triangles.

Rule

Example

Exercise

OCR GCSE Stages 5 & 6 (2007)

Giving up on meaning…

T: A fraction comes in two bits - top and bottom. Yes? The top is called numerator and the bottom denominator. The denominator means that you split it into that number of parts. The numerator tells you how many parts.... Can you all add 1/2 to 1/3?

L: You add the tops and the bottoms.

L: You times them.

T: You can't just add them together. You have to use equivalence. I want them to both end up saying 6 at the bottom. That’s the important thing.

Giving up on meaning…

Teacher writes: $\mathbf{\frac{1}{2}}\left(=\frac{}{\mathbf{6}}\right)+\mathbf{\frac{1}{3}}\left(=\frac{}{\mathbf{6}}\right)$

T: What two numbers should I choose here?

T: How many twos in six?

L: 3

T: How many threes in six?

L: 2

Teacher writes: $\mathbf{\frac{1}{2}}\left(=\mathbf{\frac{3}{6}}\right)+\mathbf{\frac{1}{3}}\left(=\mathbf{\frac{2}{6}}\right)$

T: So 3/6 + 2/6 = 5/6. Now try this one: 1/2 + 1/3. James?

L: I've not got a clue.

Giving up on meaning…

T: Let me take you back to where we don't care why we did it, I'll just tell you how. You can forget the why if you want and just remember the how. This is the way I was taught….

L: Why didn't you tell us that before?

T: Because there is just a chance that you might understand why one day.

L: How will this help us when we get older?

T: Don't ever ask me that. Its to get a grade C at the end of the year and then you'll be sure to get a good job. The whole thing about maths is its logical thinking.

4c+5p

Me:	What do you think the *c* means?
Student:	A cabbage.
Me:	In algebra could letters mean anything else?
Student:	A chair or a car…
Me:	Could the letter ever stand for a number?
Student:	No…If it was 9, then this 4*c* would be 49?

Most common learning strategies.

Statements are rank ordered from most common to least common **1 = almost never, 2 = occasionally, 3 = half the time, 4= most of the time; 5 = almost always. Source: Swan (2005)**	Mean (n=779)
I listen while the teacher explains.	**4.28**
I copy down the method from the board or textbook.	**4.15**
I only do questions I am told to do.	**3.88**
I work on my own.	**3.72**
I try to follow all the steps of a lesson.	**3.71**
I do easy problems first to increase my confidence.	**3.58**
I copy out questions before doing them.	**3.57**
I practice the same method repeatedly on many questions.	**3.42**
I ask the teacher questions.	**3.40**
I try to solve difficult problems in order to test my ability.	**3.32**
When work is hard I don't give up or do simple things.	**3.32**

Least common learning strategies

Statements are rank ordered from most common to least common **1 = almost never, 2 = occasionally, 3 = half the time, 4= most of the time; 5 = almost always. Source: Swan (2005)**	Mean (n=779)
I discuss my ideas in a group or with a partner.	**3.25**
I try to connect new ideas with things I already know.	**3.20**
I am silent when the teacher asks a question.	**3.16**
I memorise rules and properties.	**3.15**
I look for different ways of doing a question.	**3.14**
My partner asks me to explain something.	**3.05**
I explain while the teacher listens.	**2.97**
I choose which questions to do or which ideas to discuss.	**2.54**
I make up my own questions and methods.	**2.03**

Why is transmission teaching so common?

Time pressures	" It's a gallop to the main exam." " Students will waste time in social chat."
Control	" What will other teachers think of the noise?" " How can I possibly monitor what is going on?"
Views of learners	" My students cannot discuss." " My students are too afraid of being seen to be wrong."
Views of mathematics	" In mathematics, answers are either right or wrong – there is nothing to discuss." " If they understand it there is nothing to discuss. If they don't, they are in no position to discuss anything."
Views of learning	" Mathematics is a subject where you listen and practise." " Mathematics is a private activity."

‘Collaborative’, challenging culture

- **Mathematics** is seen as
  - a network of ideas which teacher and students construct together
- **Learning** is seen as
  - a social activity in which students are challenged and arrive at understanding through discussion
- **Teaching** is seen as
  - non-linear dialogue in which meanings and connections are explored
  - recognising misunderstandings, making them explicit and learning from them

..also called “Connectionist”

Drawing connections…

Time
1.43 am   1.43 pm
01.43   13.43
17 minutes before 2

1:43

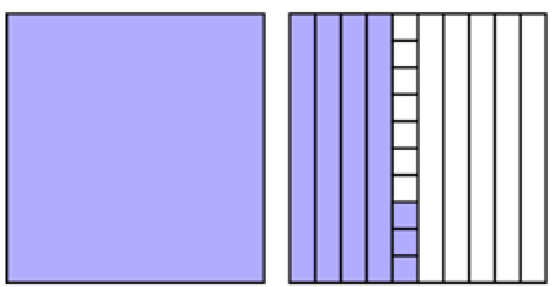
1 unit
4 tenths
3 hundredths

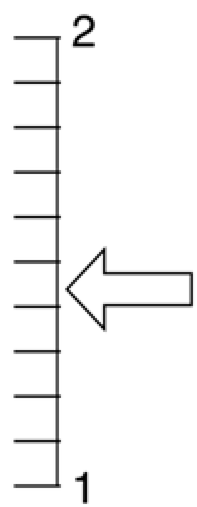
One point four three

1.43

Height
1m 43 cm
Height of a person
57 cm short of 2m

Money
£1.43
57p change from £2





Research objectives

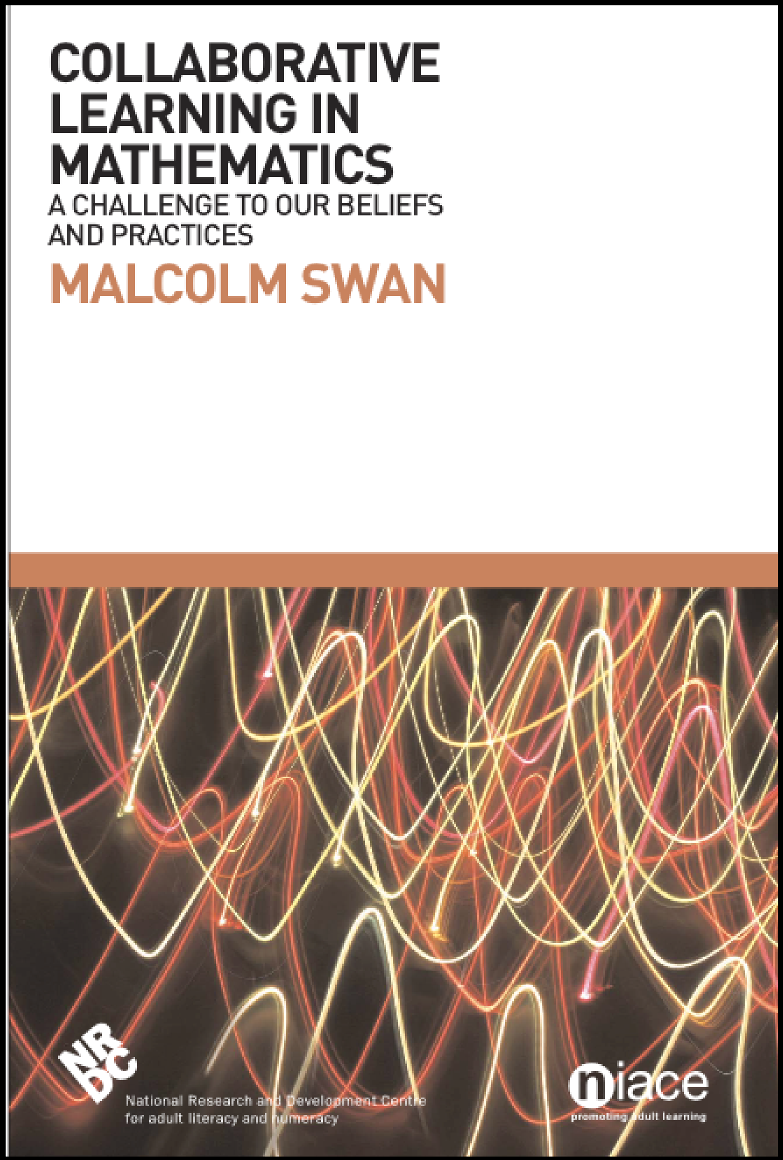
Challenge:

Can we design classroom experiences & professional development that will transform:

- **‘Transmission’ into ‘Collaborative’?**
- **‘Passive into ‘Active’?**

and create coherence between the purposes and methods of teaching?

The research background



www.mathshell.com

What types of learning do we value?

- **Fluency**
  in recalling facts, performing skills
- **Interpretations**
  for concepts and representations
- **Strategies**
  for investigation and problem solving
- **Awareness**
  of nature & values of the educational system
- **Appreciation**
  of the power of mathematics in society

Educators want a change in emphasis

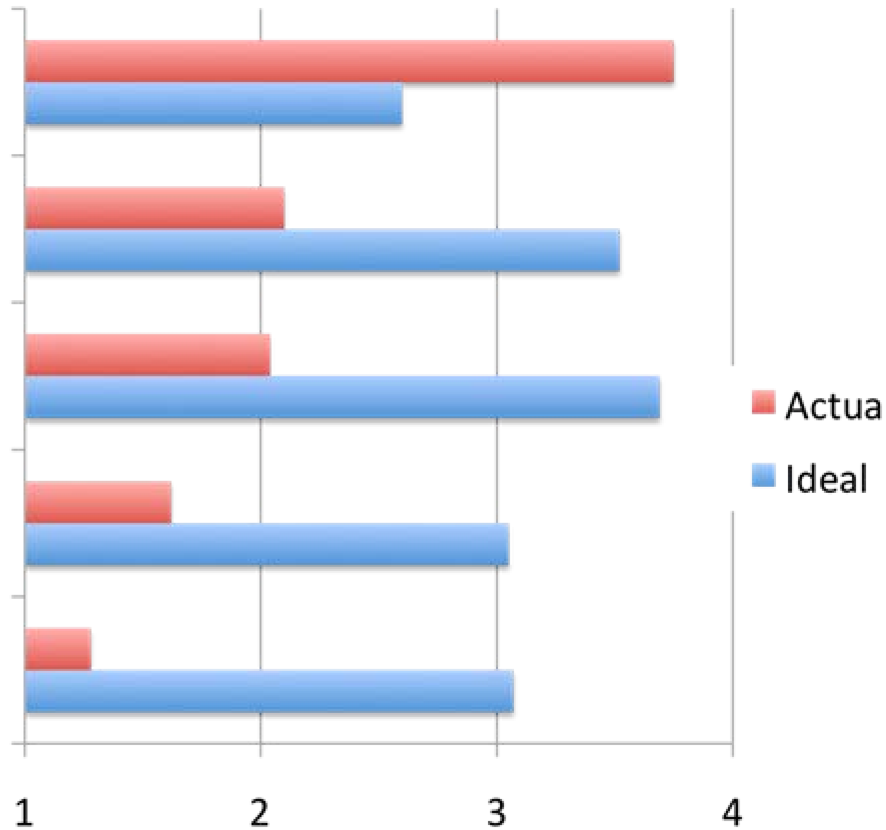
Fluency
in facts and skills

Interpretations
for concepts and representations

Strategies
for problem solving

Awareness
of educational system

Appreciation
of the power of maths in society



4 = almost all; 3 = most; 2 = less than half; 1 = few maths lessons
n=133
(Mathematics Matters, 2008)

Learning outcome	**Examples of types of activity implied**
Fluency in recalling facts & performing skills	• Memorising names and notations • Practising algorithms and procedures for fluency
Interpretations for concepts & representations	• Evaluating mathematical statements, “misconceptions” • Interpreting multiple representations • Classifying mathematical objects • Creating and solving problems, exploring structure
Strategies for investigation & problem solving	• Formulating situations and problems • Analysing, selecting and refining strategies • Monitoring progress • Interpreting and evaluating solutions and strategies • Communicating results

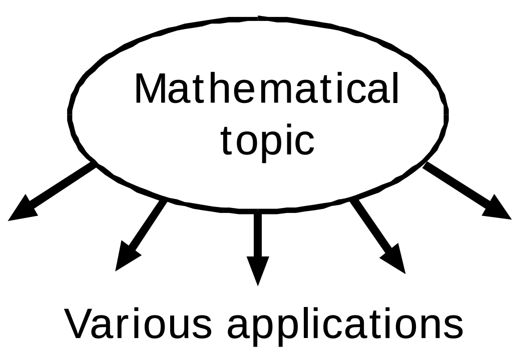
Learning outcome	Examples of types of activity implied
Awareness of values, learning and the educational system	• Discussing different purposes of learning mathematics • Developing strategies for learning and reviewing mathematics • Understanding and appreciating aspects of performance valued by the examination system
Appreciation of the power of maths in society	• Reading about cultural, historical aspects of mathematics • Creating and critiquing 'mathematical models' of situations • Discussing uses/abuses of mathematics in social contexts • Solving problems in one's own life

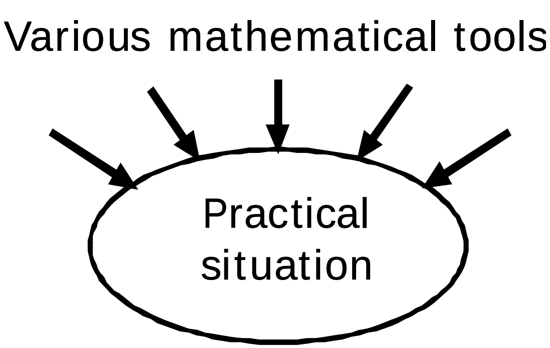
Tensions and conflicts

- Content coverage (Faster.. Faster)
- Reflection and creativity (Take you time…)

- Convergence on important theorems (Learn this)
- Openness of investigational work (Explore this)

- Illustrative applications (Use this method)
- Real life problem solving (Use any method)





Empirical studies “Diagnostic teaching”

- **Expose existing ideas and concepts**

- **Confront with implications and contradictions**
  provoke ‘tension’ or ‘cognitive conflict’

- **Resolve conflict through discussion**
  encouraging the formulation of new concepts and methods.

- **Generalise, extend and link learning**
  applying the new concepts and methods on further problems.

Analogy from screenplay writing

- **Set-up:** This part introduces the characters their relationships and their world. It sets up a dramatic premise, situation and question that will be answered at the climax.
  “ Will X get the girl?” “Will Y get the killer?”
- **Confrontation with an obstacle**: Often this results in worsening situation. *“They must not only learn new skills but arrive at a higher sense of awareness of who they are and what they are capable of, in order to deal with their predicament. This cannot be achieved alone and they are usually aided and abetted by mentors and co-protagonists”.*
- **Resolution**: Includes a climax and a dénouement and maybe a ‘twist’.

(Trottier, David: "The Screenwriter's Bible”, 1998)

Interpreting Decimals

Put these numbers in order of size,
from smallest to largest.

Write down how you did this.

Now use the cards and work with a partner.
Try to agree.

0.8	0.04
0.25	0.375
0.4	0.125
0.75	

Interpreting Decimals

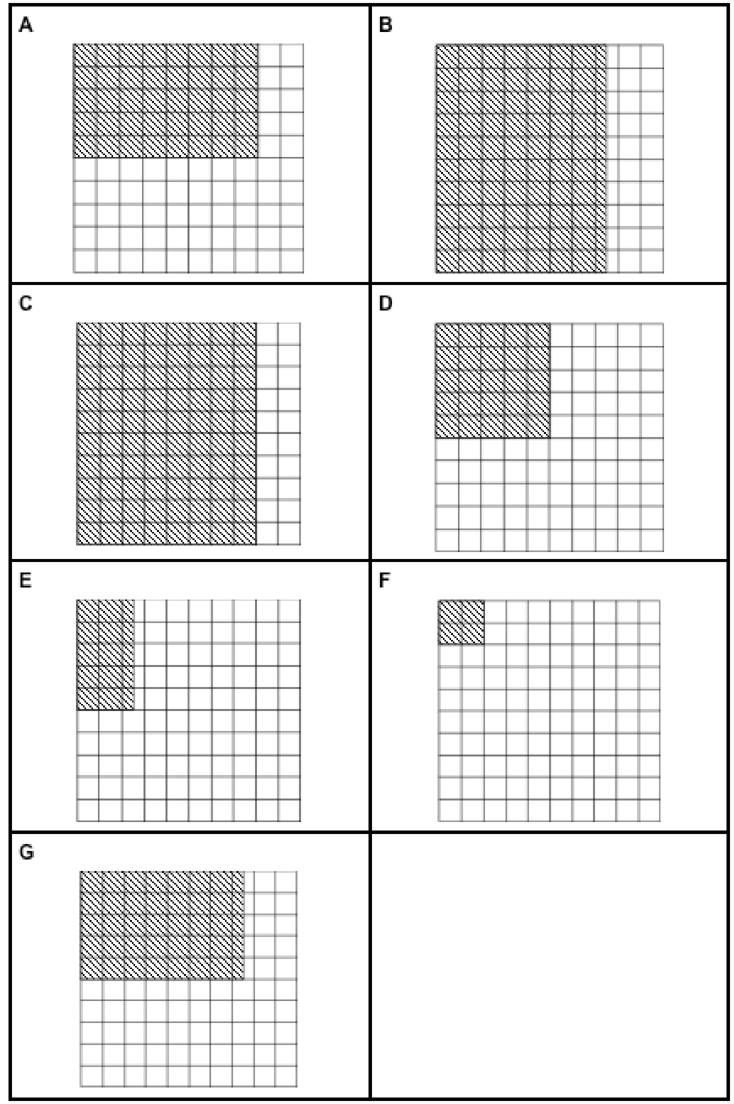
0.375	0.125	0.75	0.25	0.04	0.4	0.8

“I know this because they work like fractions, 0.8 is like an eighth.”

“The more digits there are, the smaller the decimal is.”

Interpreting Decimals

0.8	0.04
0.25	0.375
0.4	0.125
0.75	



Interpreting Fractions

Put these numbers in order of size,
from smallest to largest.

Write down how you did this.

Now use the cards and work with a partner.
Try to agree.

$\frac{3}{8}$	$\frac{2}{5}$
$\frac{1}{4}$	$\frac{3}{4}$
$\frac{8}{10}$	$\frac{1}{25}$
$\frac{1}{8}$	

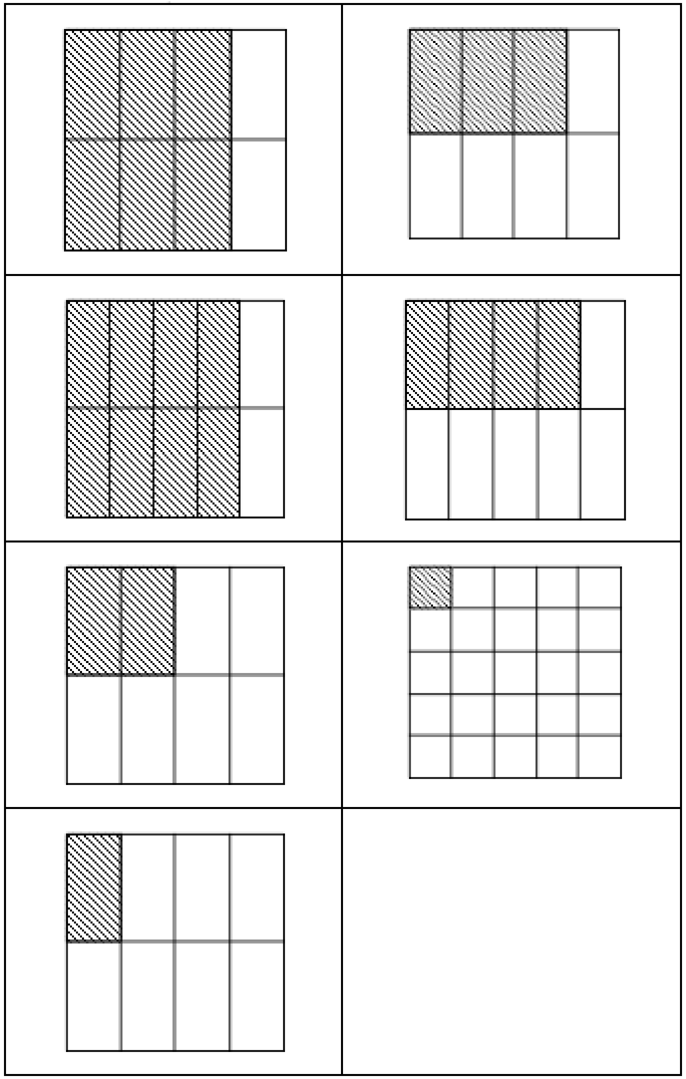
Interpreting Fractions

$\frac{1}{25}$	$\frac{8}{10}$	$\frac{1}{8}$	$\frac{3}{8}$	$\frac{2}{5}$	$\frac{1}{4}$	$\frac{3}{4}$

"Because 1/25 is much smaller than 3/4. If you cut the fraction out of a cake there would be 25 small pieces instead of 4 large pieces."

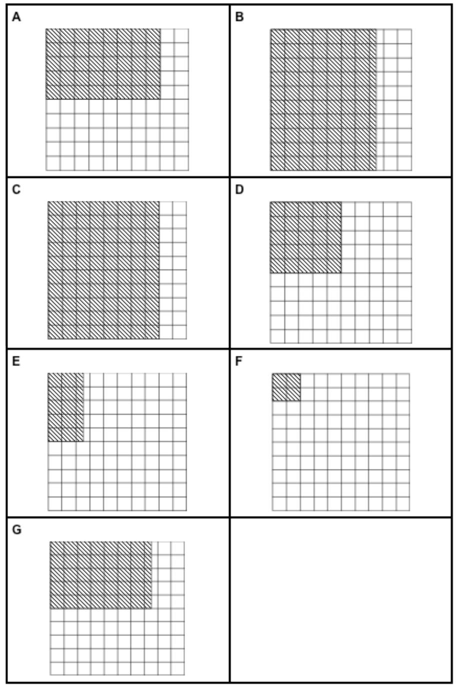
Interpreting Fractions

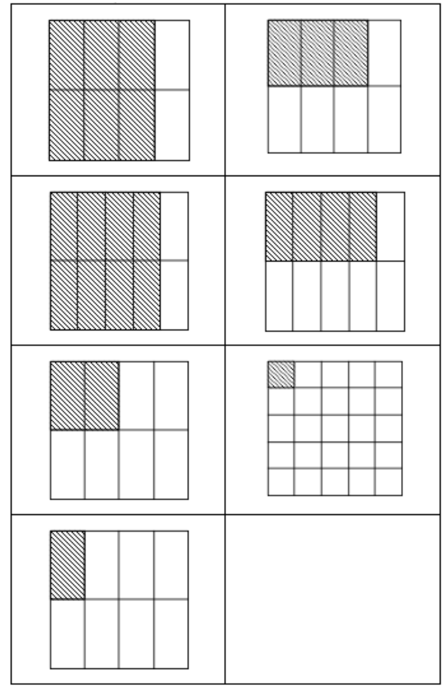
$\frac{3}{8}$	$\frac{2}{5}$
$\frac{1}{4}$	$\frac{3}{4}$
$\frac{8}{10}$	$\frac{1}{25}$
$\frac{1}{8}$	



Linking fractions and decimals

0.8	0.04
0.25	0.375
0.4	0.125
0.75	

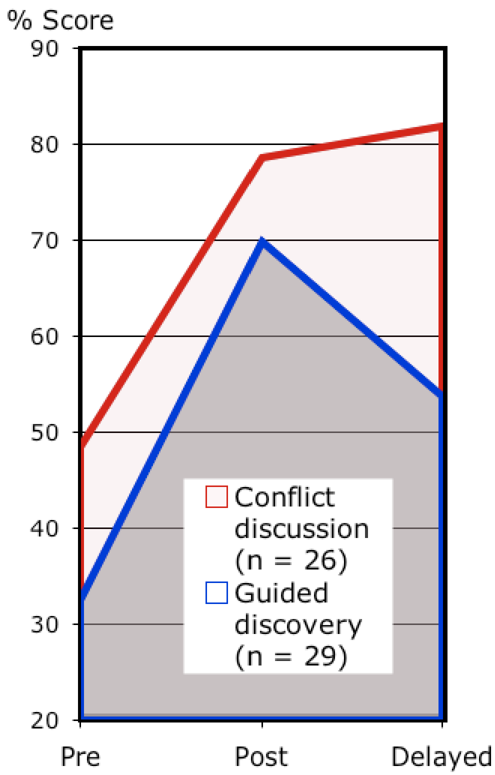




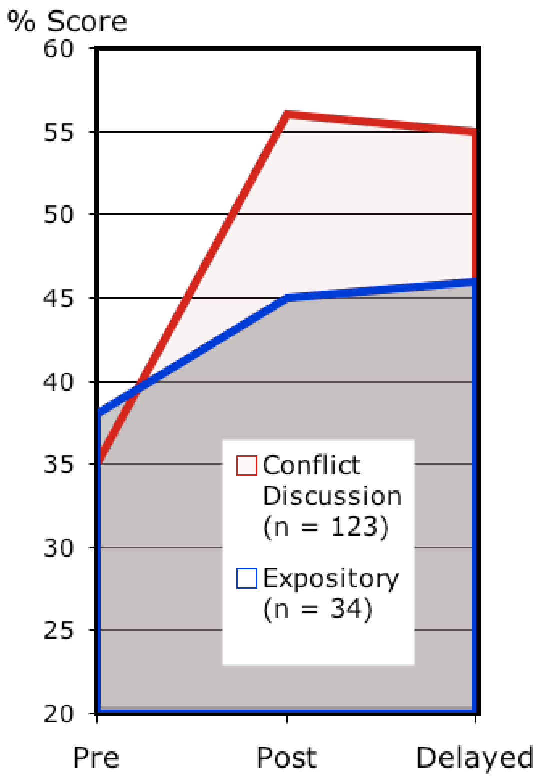
$\frac{3}{8}$	$\frac{2}{5}$
$\frac{1}{4}$	$\frac{3}{4}$
$\frac{8}{10}$	$\frac{1}{25}$
$\frac{1}{8}$	

‘Diagnostic Teaching’ Research

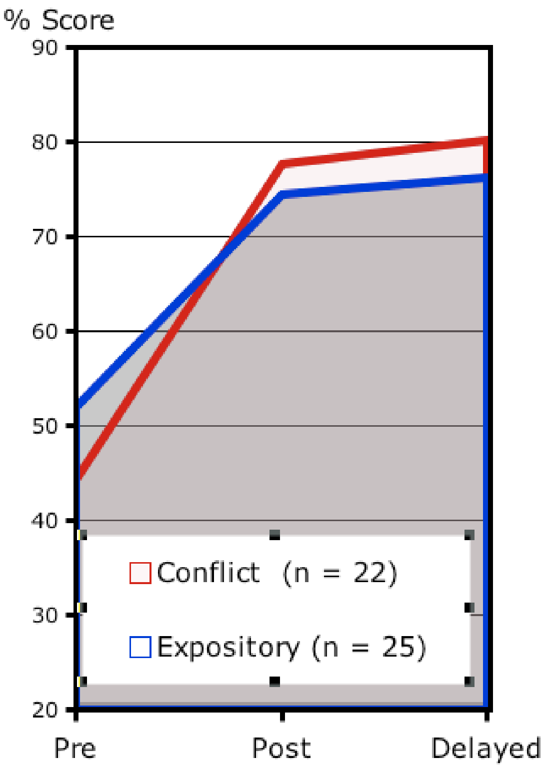
Reflections



Rates



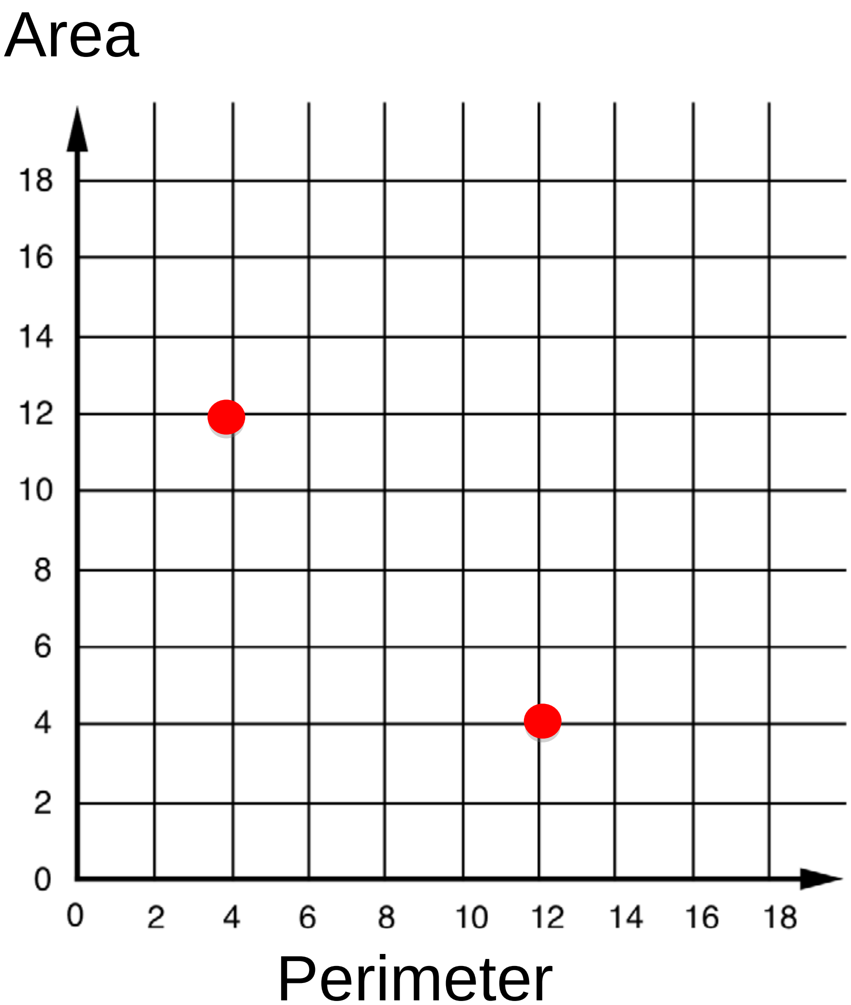
Decimals



Principles for effective teaching

- **Build on knowledge students already have.**
- **Expose and discuss misconceptions.**
- **Use cooperative small group work.**
- **Use rich collaborative tasks.**
- **Create connections between topics.**
- **Use higher-order questions.**
- **Emphasise reasons rather than answers.**
- **Use technology in appropriate ways.**

Rich tasks to promote reasoning



Draw a shape on squared paper and plot a point to show its perimeter and area.

Which points on the grid represent squares, rectangles …..

Draw a shape that may be represented by the point (12,4)

Draw a shape that may be represented by the point (4,12)

Find all the 'impossible' points.

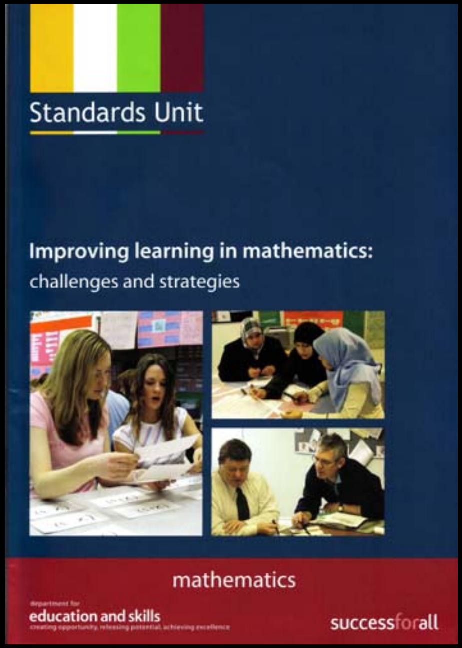
Posters to share reasoning

QuickTime™ and a
decompressor
are needed to see this picture.

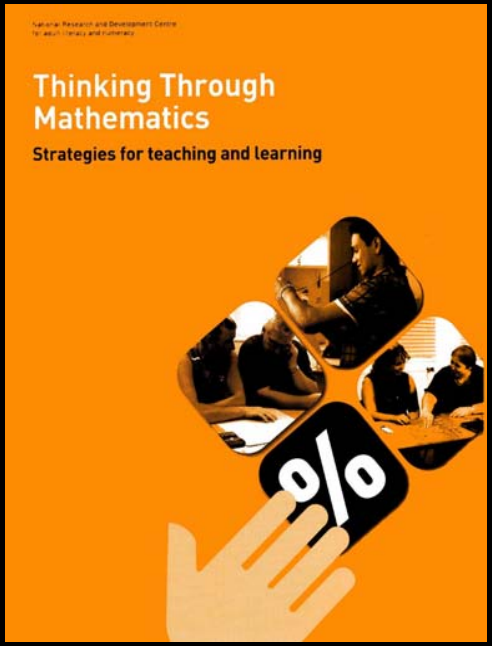
Mini-whiteboards to feed back ideas

QuickTime™ and a
decompressor
are needed to see this picture.

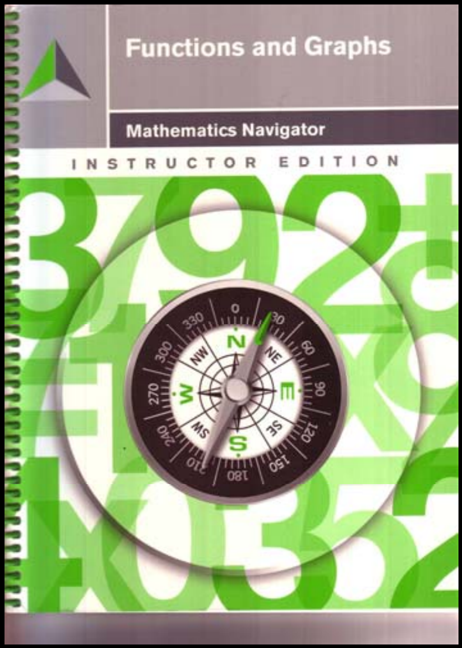
Lesson genres



Improving Learning
in Mathematics
Post-16, GCSE A level



Thinking Through
Mathematics
Adult numeracy



Mathematics
Navigator
Middle School
America's Choice
(USA)

Common elements of the materials

- **Shared student resources**
- **Lesson by lesson teaching plans**
  - *Beginning the session*
  - *Working in groups*
  - *Reviewing and extending learning*
- **DIY Professional development resources**
  - *Multimedia – classroom videos, computer software*
  - *Supporting software*
  - *Activities for teachers to work on together*

Activity 'Genres' that develop thinking…

1. Evaluating mathematical statements
2. Interpreting multiple representations
3. Classifying mathematical objects
4. Creating and solving problems...

Natural Powers

Involve learners in using their natural powers to explore situations and solve problems:

- Distinguishing and Connecting
- Organising and characterising
- Stressing and ignoring
- Specialising and generalising
- Conjecturing and convincing
- Imagining and expressing

(Mason, Johnston-Wilder, 2004)

Evaluating mathematical statements

Students…

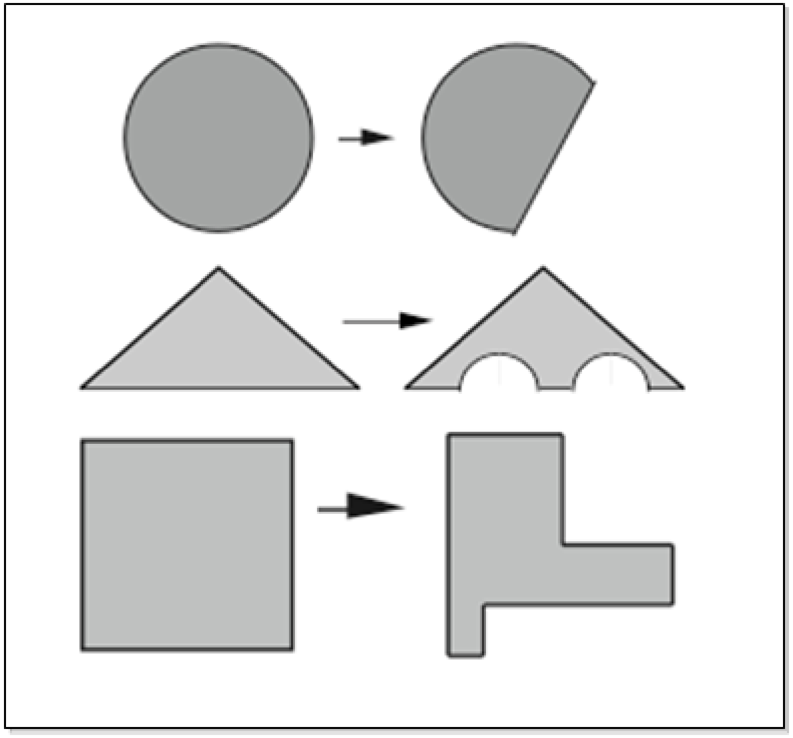
- Decide whether given statements are always, sometimes or never true.
- Develop rigorous mathematical arguments and justifications;
- Create examples and counterexamples to defend their reasoning.

Evaluating mathematical statements

Pay rise Max gets a pay rise of 30%. Jim gets a pay rise of 25%. So Max gets the bigger pay rise.	**Sale** In a sale, every price was reduced by 25%. After the sale every price was increased by 25%. So prices went back to where they started.
Area and perimeter When you cut a piece off a shape you reduce its area and perimeter	**Means** $\dfrac{a+b}{2} \geq \sqrt{ab}$
Bigger fractions If you add the same number to the top and bottom of a fraction, the fraction gets bigger in value.	**Smaller fractions** If you divide the top and bottom of a fraction by the same number, the fraction gets smaller in value.

Always, sometimes or never true?

When you cut a piece off a shape you reduce its area and perimeter

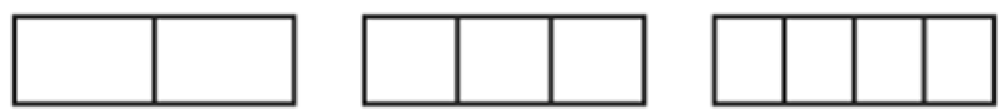


Always, sometimes or never true?

When you add the same number to the top and bottom of a fraction, the fraction gets bigger in value.

Draw diagrams to show:

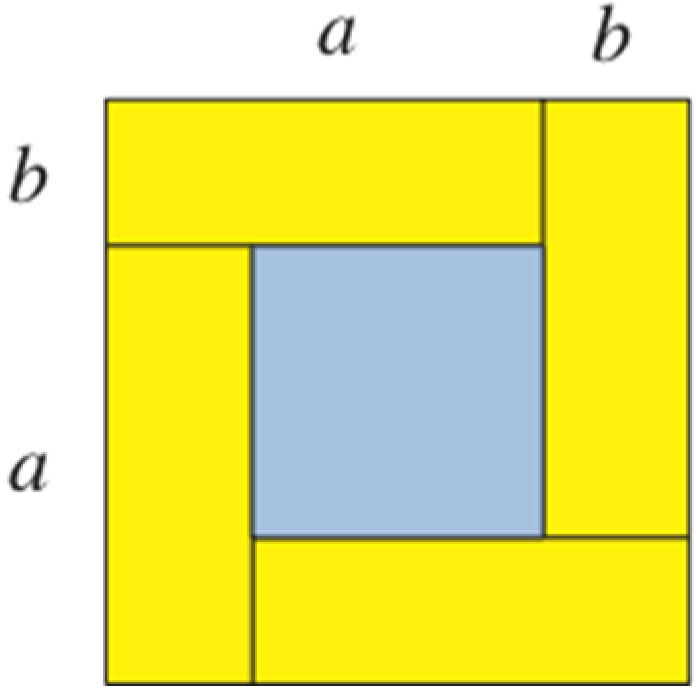
$$\frac{1}{2}, \frac{2}{3}, \frac{3}{4}, \ldots$$



What happens to the size?
Now you try a different sequence.

Always, sometimes or never true?

$$\frac{a+b}{2} \geq \sqrt{ab}$$

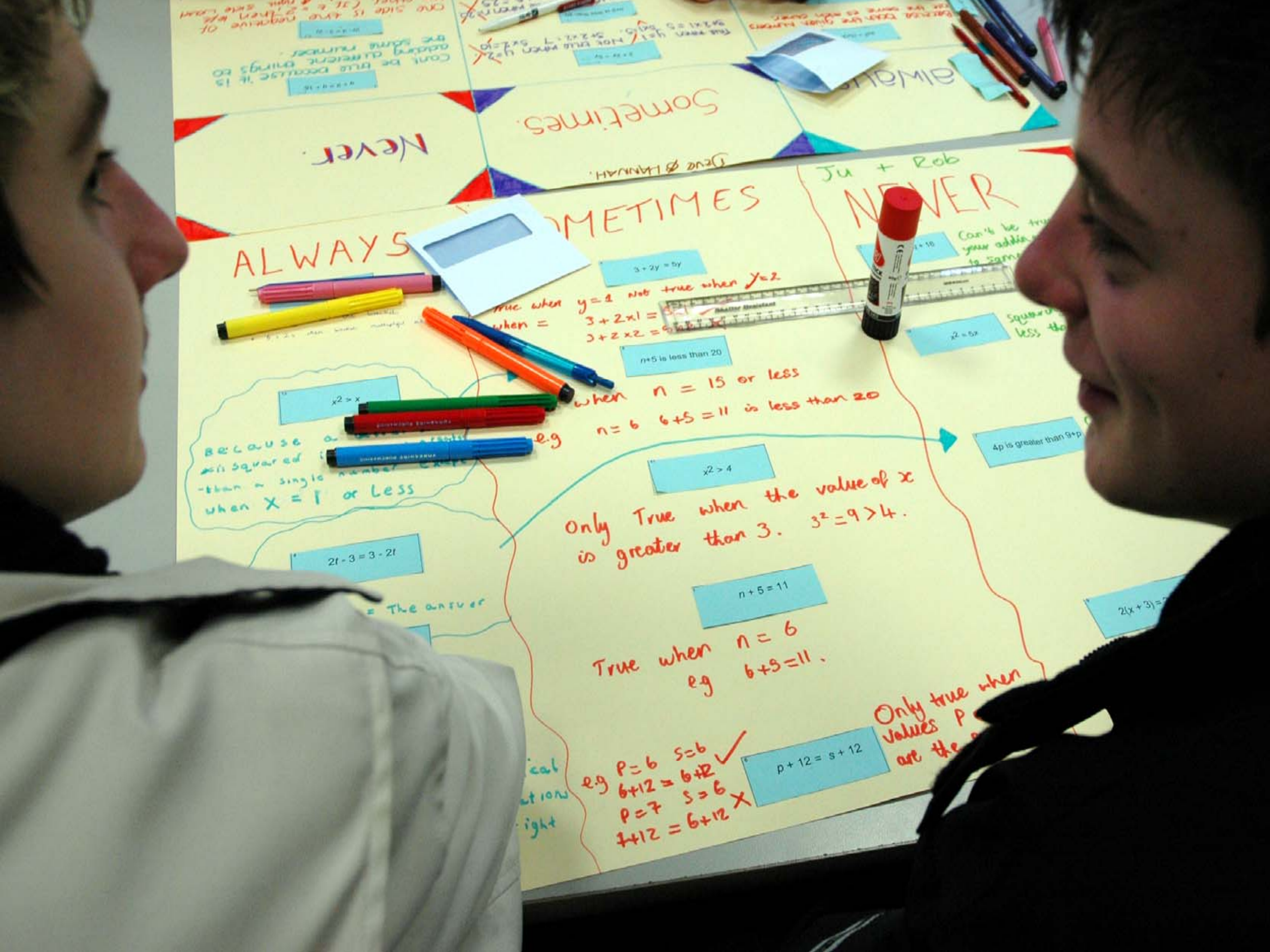


The meaning of operations: Always, sometimes or never true?

It doesn't matter which way round you multiply, you get the same answer. $a \times b = b \times a$	It doesn't matter which way round you divide, you get the same answer. $a \div b = b \div a$
If you add a number to 12 you get a number greater than 12. $12 + a > 12$	If you divide 12 by a number the answer will be less than 12. $12 \div a < 12$
The square root of a number is less than the number. $\sqrt{a} < a$	The square of a number is greater than the number. $a^2 > a$

Always,Sometimes or Never True?

$p + 12 = s + 12$	$3 + 2y = 5y$
$n+5$ is less than 20	$4p > 9+p$
$2(x + 3) = 2x + 3$	$2(3 + s) = 6 + 2s$

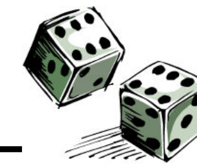


Never
Sometimes
Ju + Rob
ALWAYS
OMETIMES
3 + 2y = 5y
n+5 is less than 20
when n = 15 or less
x2 > x
x2 > 4
Only True when the value of x is greater than 3. 3² = 9 > 4.
2t - 3 = 3 - 2t
n + 5 = 11
True when n = 6 eg 6+5=11.
p + 12 = s + 12
x2 = 5x

True, false or unsure?

When you roll a fair six-sided die, it is harder to roll a six than a four.	Scoring a total of three with two dice is twice as likely as scoring a total of two.
In a lottery, the six numbers 3, 12, 26, 37, 38, 40 are more likely to come up than the numbers 1, 2, 3, 4, 5, 6.	There are three outcomes in a football match, win lose or draw. The probability of winning is therefore 1/3
If a family has already got four boys, then the next baby is more likely to be a girl than a boy.	In a ‘true or false’ quiz with ten questions, you are certain to get five right if you just guess.

True, false or unsure?



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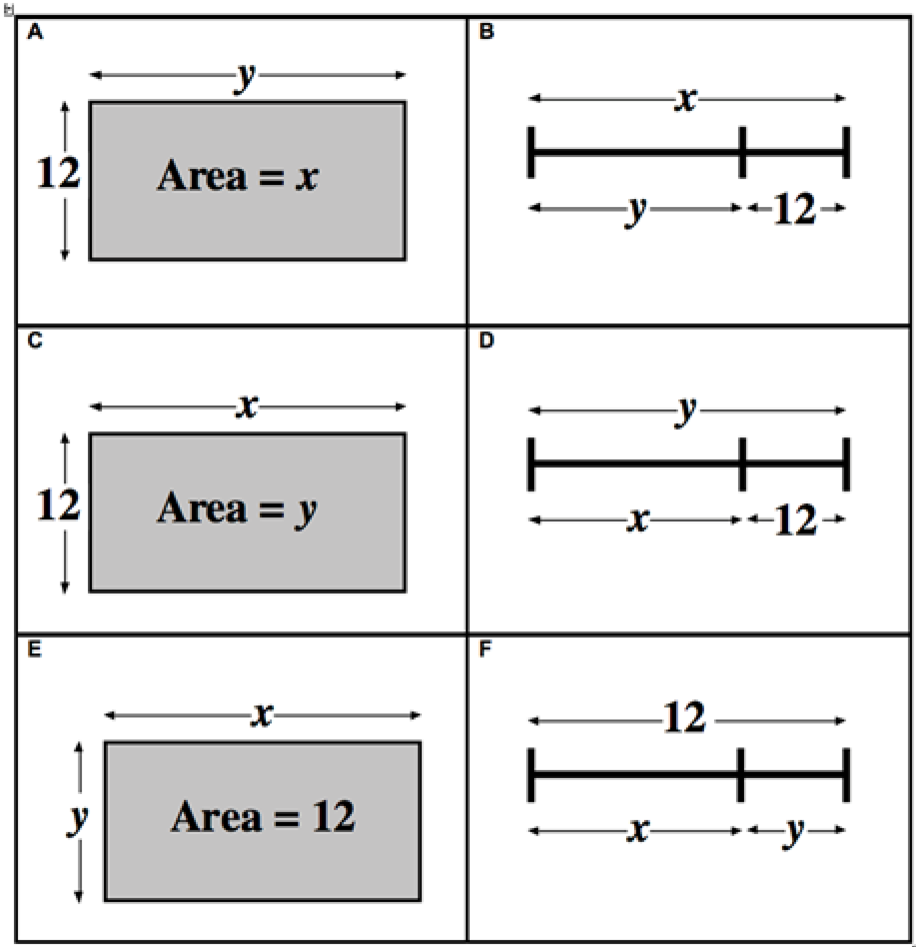
Interpreting Multiple Representations

Helps students to:

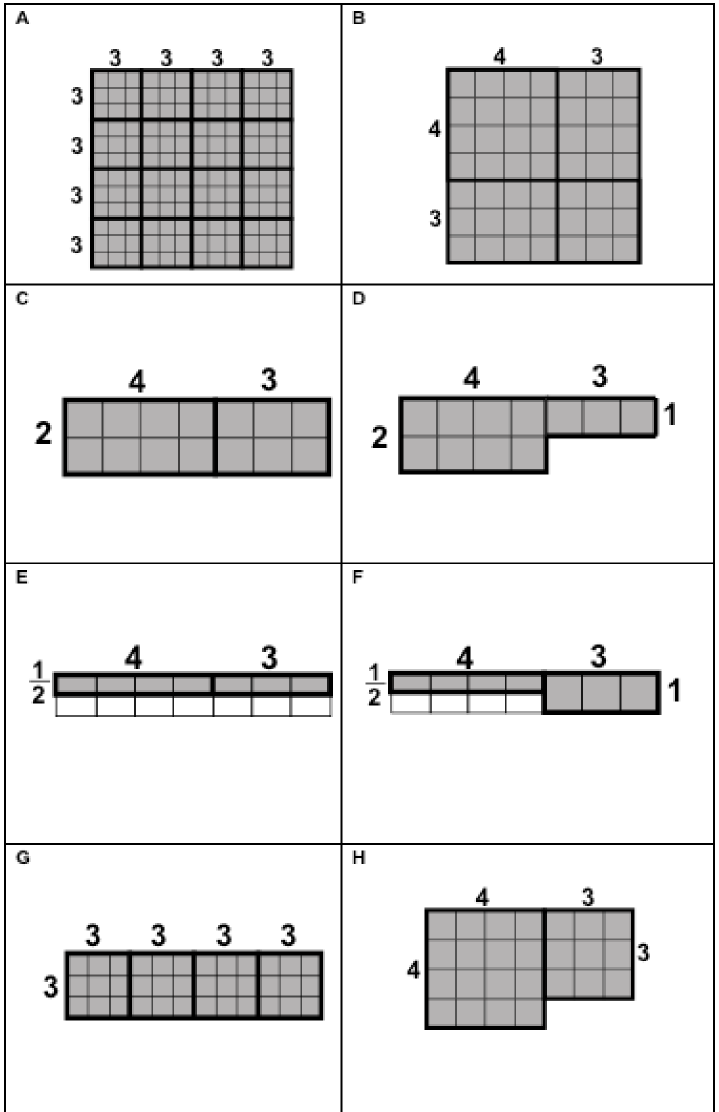
- visualise and verbalise concepts in different ways;
- make connections;
- translate between different representations of concepts.

Rearranging equations



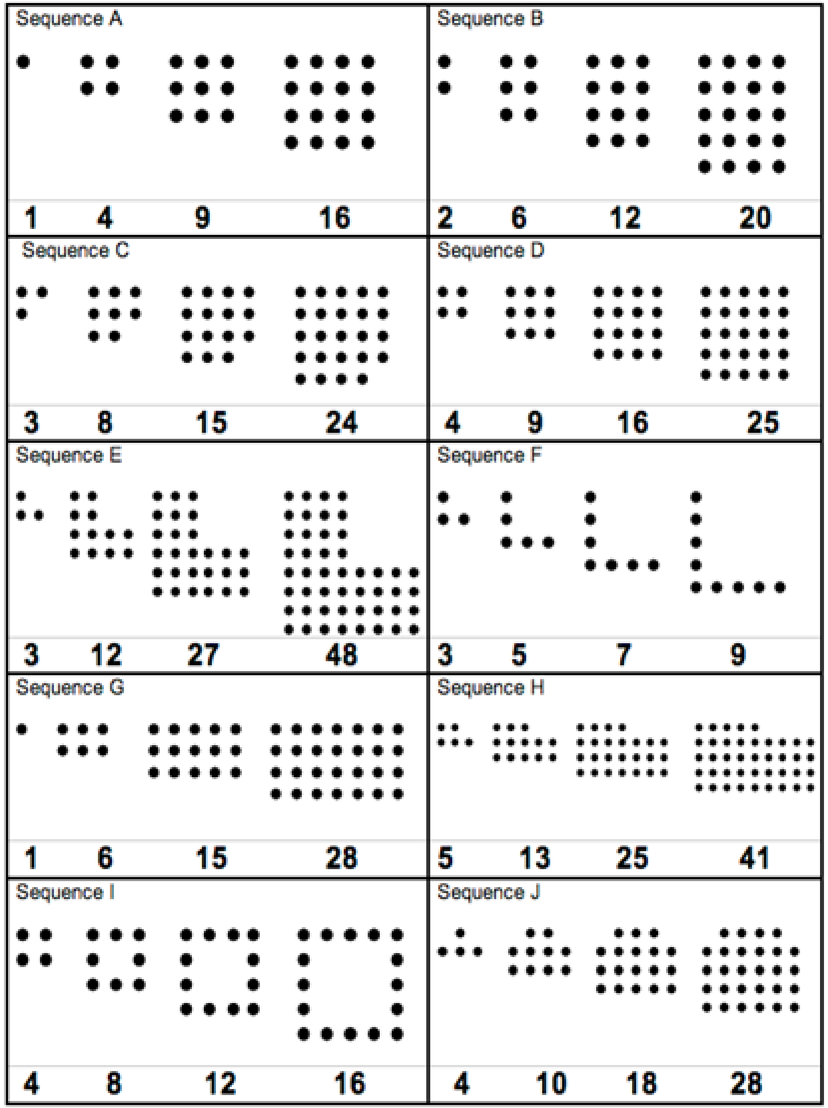
A $y = \frac{12}{x}$	B $y = x + 12$
C $y = x - 12$	D $y = 12 - x$
E $y = 12x$	F $y = \frac{x}{12}$
G $x = 12 - y$	H $x = \frac{y}{12}$
I $x = y + 12$	K $x = y - 12$
L $x = 12y$	M $x = \frac{12}{y}$

Algebraic notation

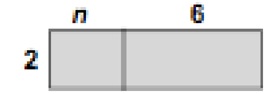
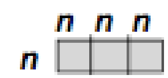


A $2\times(4+3)$	B $2\times4+3$
C $\frac{1}{2}(4+3)$	D $\frac{4+3}{2}$
E $2\times4+2\times3$	F 4×3^2
G $(4\times3)^2$	H $(4+3)^2$
I 4^2+3^2	J $4^2+2\times3\times4+3^2$
K $\frac{4}{2}+\frac{3}{2}$	L $\frac{1}{2}\times4+3$
M $4^2\times3^2$	N $3+4\times2$

Expressing sequences in different ways



n^2	$4n$
n^2+2n+1	n^2+2n
n^2+n	$2n+1$
$2n^2-n$	$3n^2$
n^2+3n	$(n+1)^2$
$n(n+1)$	$n(2n-1)$
$(n+1)^2-1$	$(n+1)^2-n^2$
$(2n)^2-n^2$	$(n+1)^2-(n-1)^2$
$n+n(n+2)$	$n^2+(n+1)^2$
$(n+1)(2n+1)-n$	$n+n(n-1)$

<table>
<tr><td>$\dfrac{n+6}{2}$</td><td>$3n^2$</td><td>Square n, then multiply by three</td><td><table><tr><td>n</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>Ans</td><td>14</td><td>16</td><td>18</td><td>20</td></tr></table></td><td>
</td></tr>
<tr><td>$2n+12$</td><td>$2n+6$</td><td>Add six to n, then multiply by two.</td><td><table><tr><td>n</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>Ans</td><td></td><td></td><td>81</td><td>144</td></tr></table></td><td>
</td></tr>
<tr><td>$2(n+3)$</td><td>$\dfrac{n}{2}+6$</td><td>Add six to n, then divide by two</td><td><table><tr><td>n</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>Ans</td><td></td><td>10</td><td>15</td><td>22</td></tr></table></td><td>
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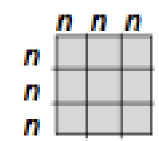
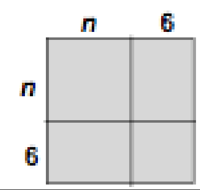
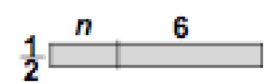
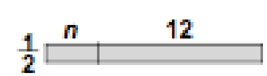
<tr><td>$(3n)^2$</td><td>$(n+6)^2$</td><td>Divide n by two, then add three</td><td><table><tr><td>n</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>Ans</td><td>3</td><td></td><td>27</td><td>48</td></tr></table></td><td>
</td></tr>
<tr><td>$n^2+12n+36$</td><td>$\dfrac{n}{2}+3$</td><td>Add six to n, then square the answer</td><td><table><tr><td>n</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>Ans</td><td></td><td></td><td>81</td><td>100</td></tr></table></td><td>
</td></tr>
<tr><td>n^2+6</td><td>Add three to n then multiply by two.</td><td>Square n, then multiply by nine</td><td><table><tr><td>n</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>Ans</td><td></td><td>4</td><td></td><td>5</td></tr></table></td><td>
</td></tr>
<tr><td>n^2+6^2</td><td>Multiply n by two then add twelve</td><td>Multiply n by two, then add six.</td><td><table><tr><td>n</td><td>1</td><td>2</td><td>3</td><td>4</td></tr><tr><td>Ans</td><td>6.5</td><td>7</td><td>7.5</td><td>8</td></tr></table></td><td>
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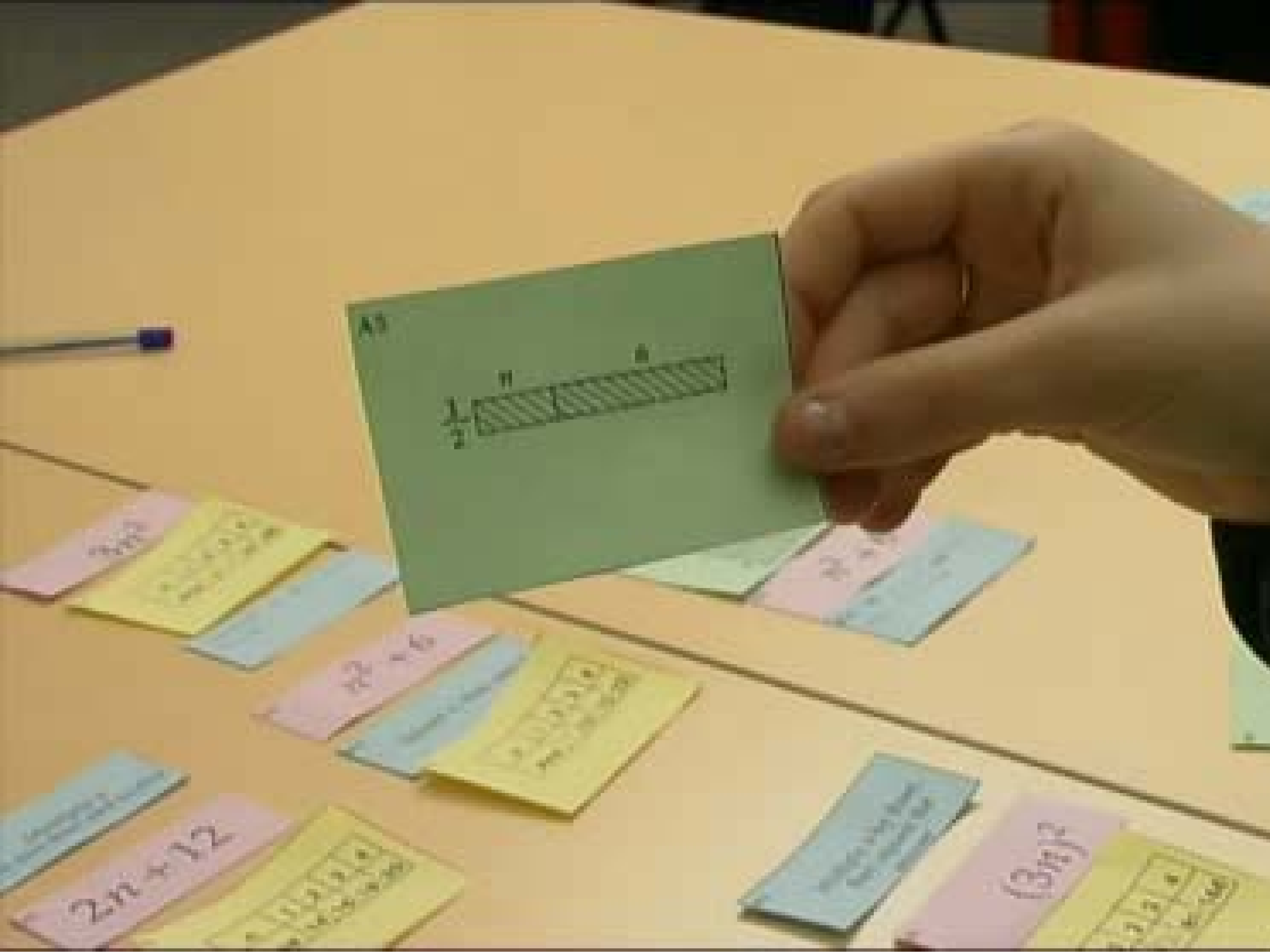
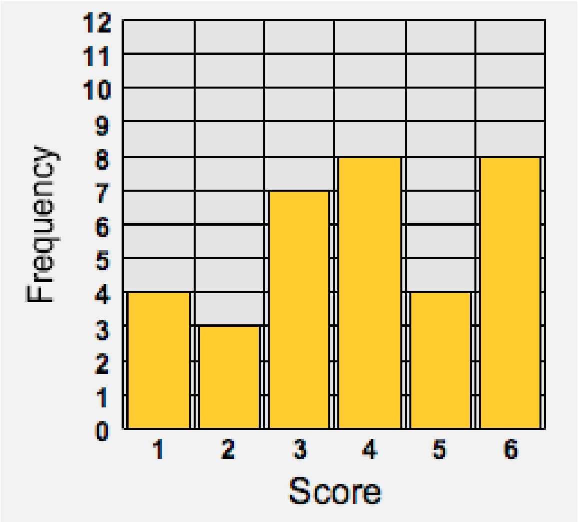
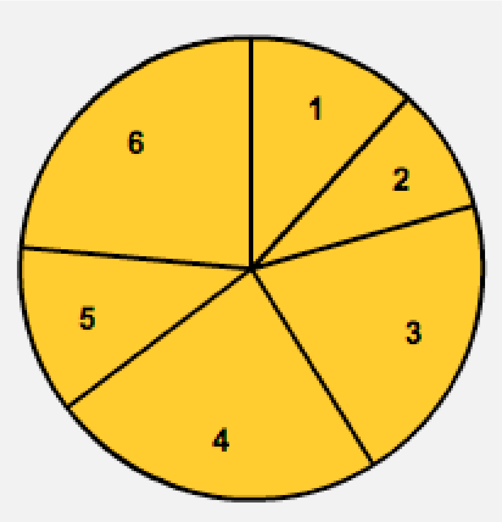
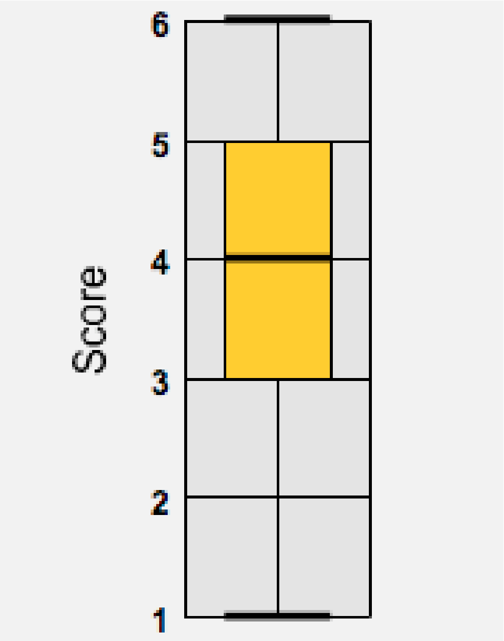

A5
2n + 12

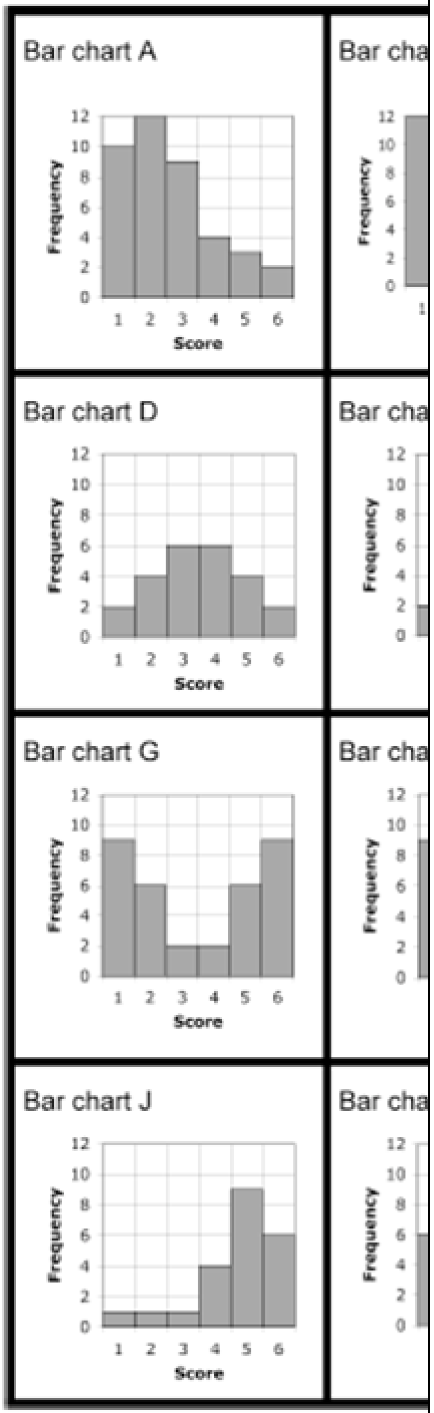
Penalty kicks

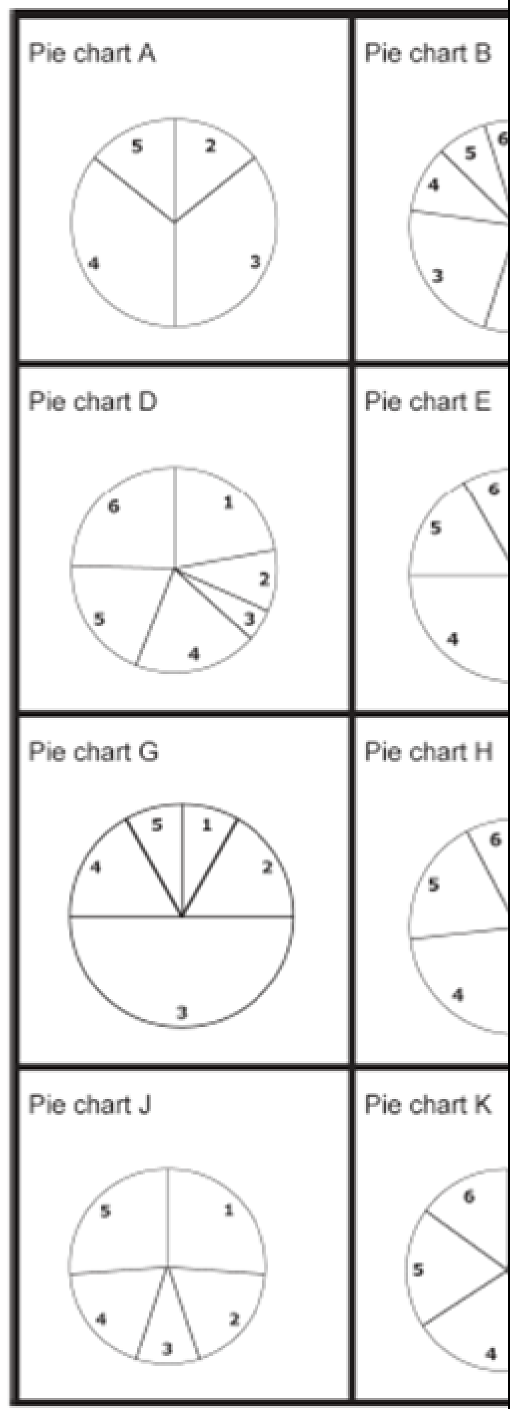
These charts represents the scores that were obtained when a number of people entered a penalty-taking competition. Each person was allowed six penalty kicks.

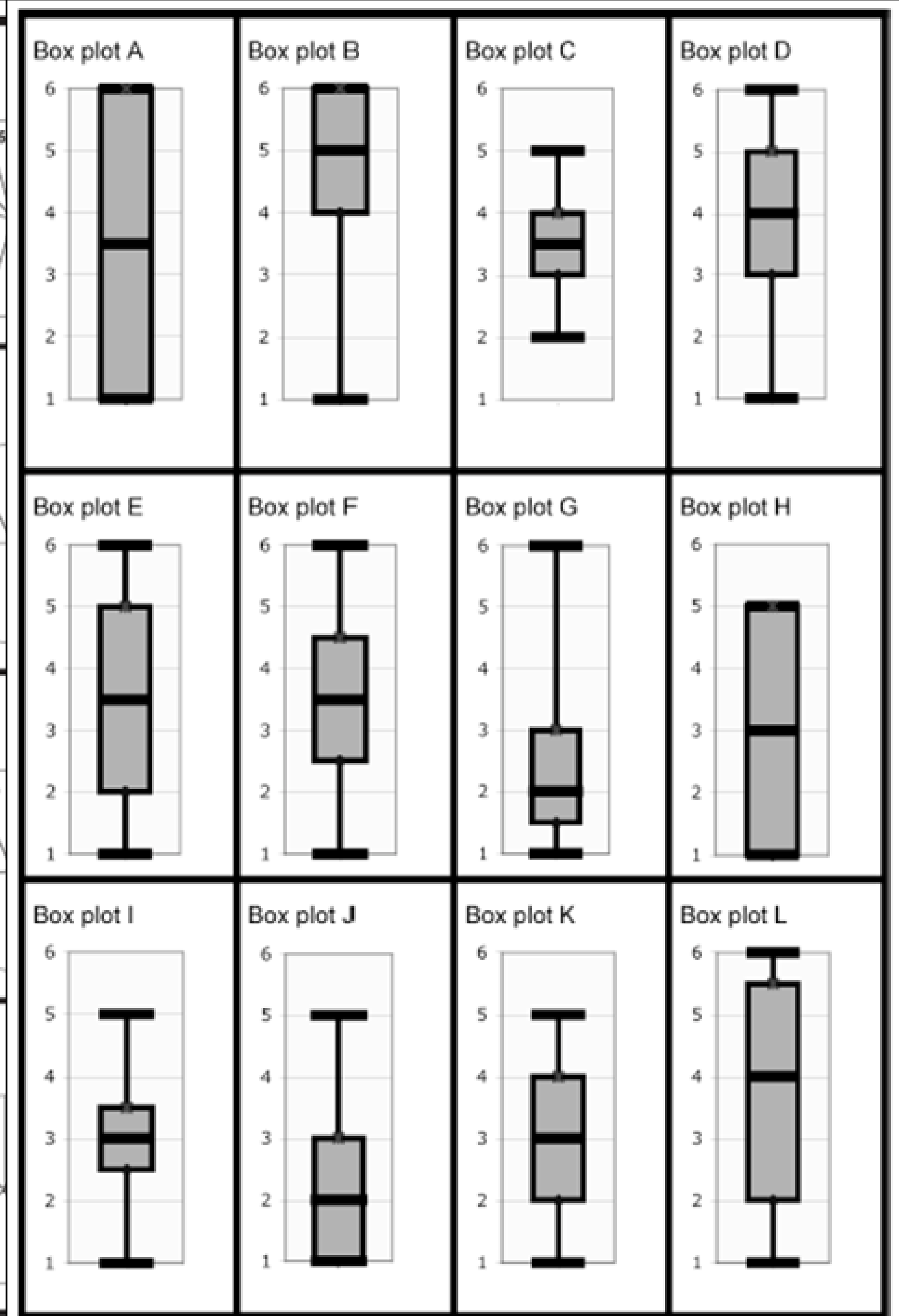







Bar chart A
Bar chart D
Bar chart G
Bar chart J
Frequency
Score


Pie chart A
Pie chart B
Pie chart D
Pie chart E
Pie chart G
Pie chart H
Pie chart J
Pie chart K


Box plot A
Box plot B
Box plot C
Box plot D
Box plot E
Box plot F
Box plot G
Box plot H
Box plot I
Box plot J
Box plot K
Box plot L

Change the **frequency table** and watch what happens

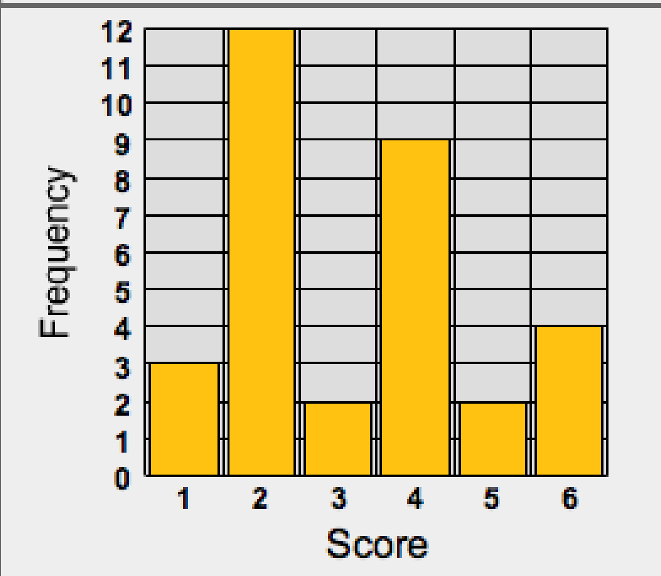
Frequency Table Hide [F]

Score	1	2	3	4	5	6
Frequency	3	12	2	9	2	4

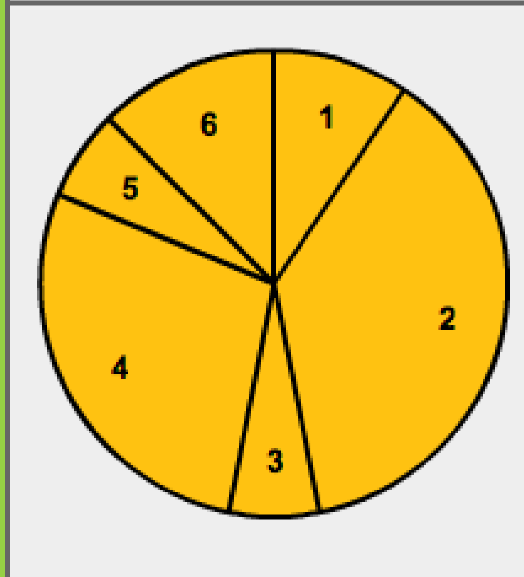
Statistics Hide [S]

Mean	3.22
Median	3
Mode	2
Range	5

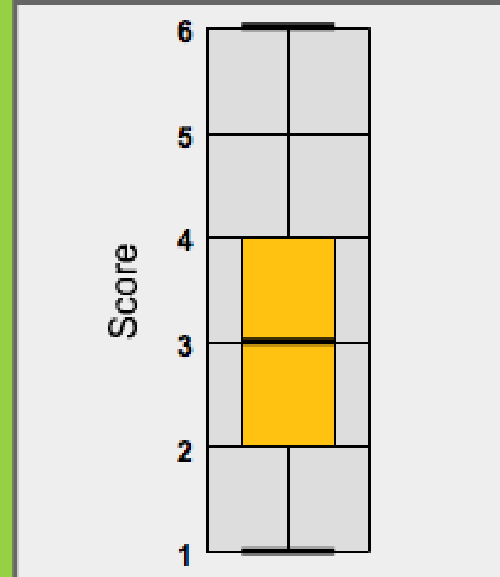
Bar chart Hide [B]



Pie chart Hide [P]



Box & Whisker Hide [W]

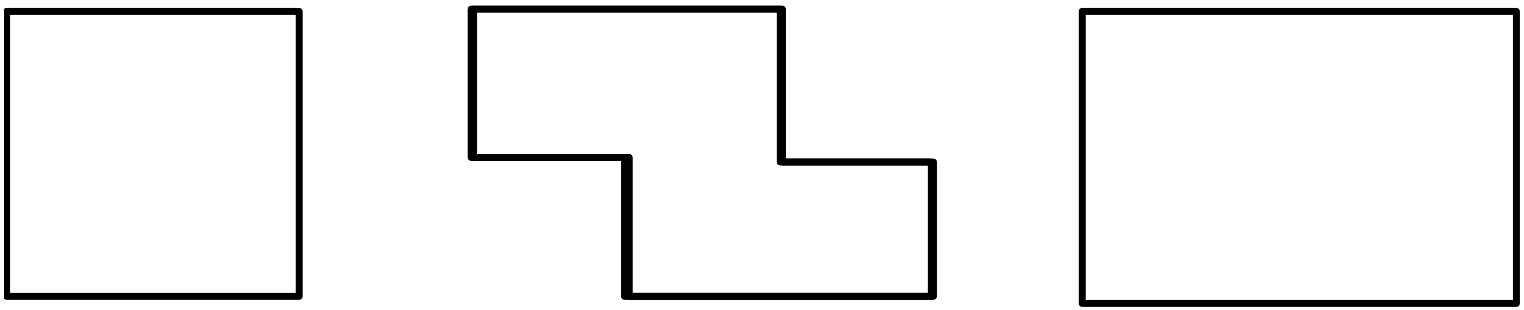


Classifying mathematical objects

Students learn to:

- Discriminate carefully
- Recognise properties of objects
- Classify mathematical objects according to different attributes
- Create and use categories to build definitions
- Develop mathematical language

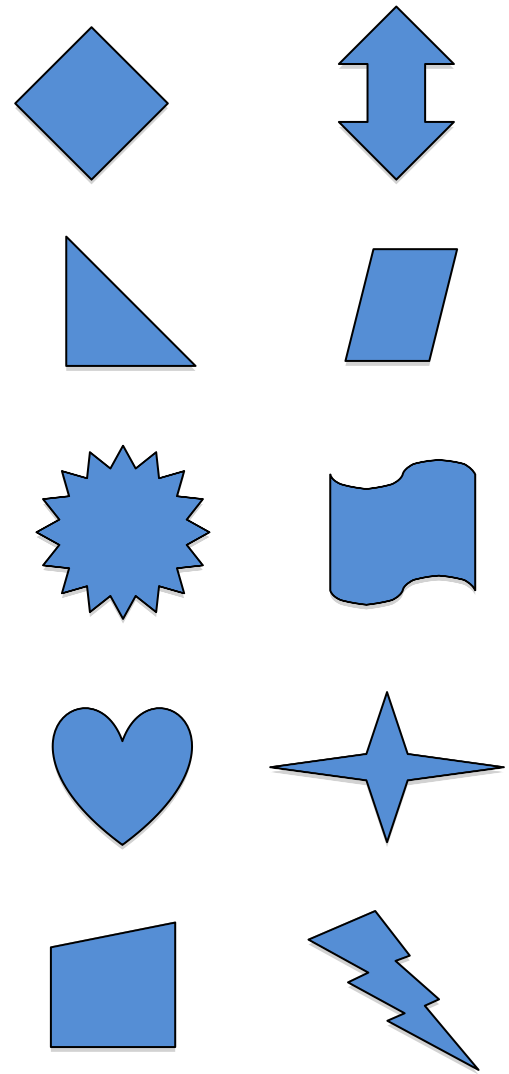
Classifying:
Why might each be the ‘odd one out’?

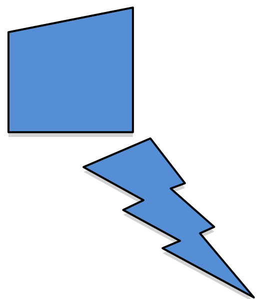
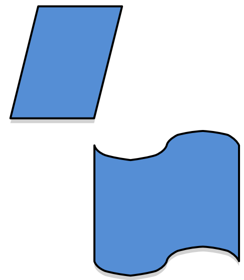
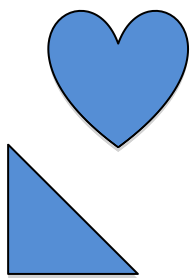
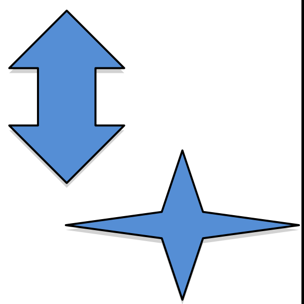
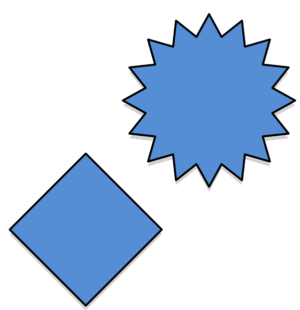


$\frac{3}{4}$ $\frac{2}{7}$ $\frac{1}{10}$

$y = x^2 - 6x + 8$ $y = x^2 - 6x + 9$ $y = x^2 - 6x + 10$

	No rotational symmetry	Rotational symmetry
No lines of symmetry		
One or two lines of symmetry		
More than two lines of symmetry		



	No rotational symmetry	Rotational symmetry
No lines of symmetry		
One or two lines of symmetry		
More than two lines of symmetry		

Is it possible to find a shape that has no rotational symmetry which has more than two lines of symmetry?

$y = x^2 + 2x + 4$

$y = 4x^2 - 4x + 1$

$y = x^2 - 5x + 4$

$y = x^2 - 4x + 4$

$y = x^2 + 7x - 3$

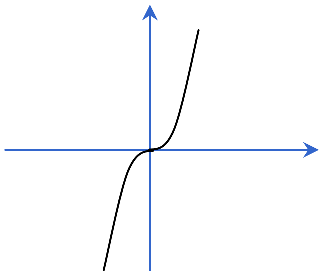
$y = 2x^2 - 5x - 3$

	Factorises with integers	Does not factorise with integers
Two x intercepts		
One x intercept		
No x intercepts		

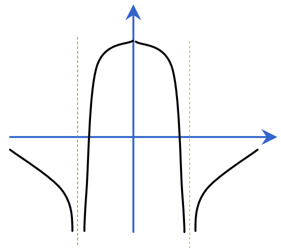
Is it possible to find a quadratic function y=f(x) that factorises but has no x intercepts?

	Factorises with integers	**Does not factorise with integers**
Two x intercepts	$y = x^2 - 5x + 4$	$y = 2x^2 - 5x - 3$ $y = x^2 + 7x - 3$
One x intercept	$y = x^2 - 4x + 4$	$y = 4x^2 - 4x + 1$
No x intercepts		$y = x^2 + 2x + 4$

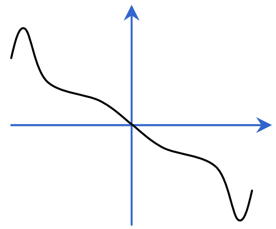
How would you classify these graphs?



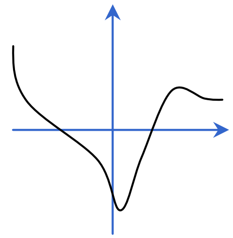
(1)



(2)



(3)



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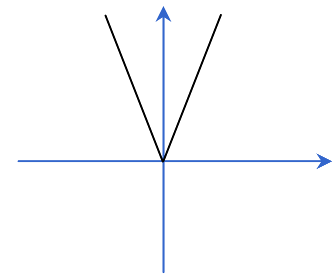

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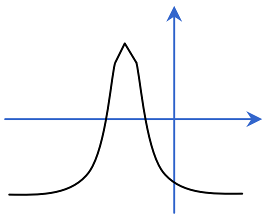

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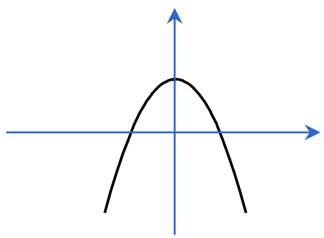

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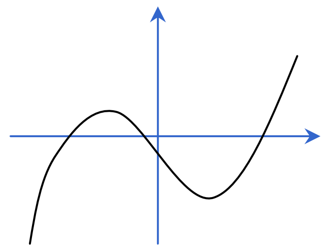

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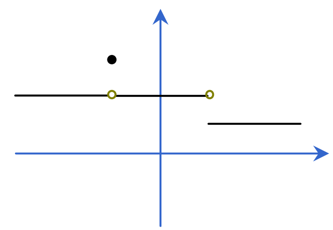

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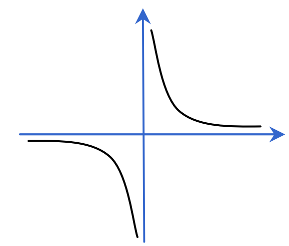

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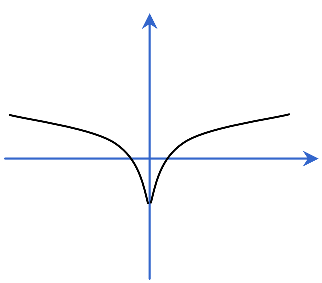

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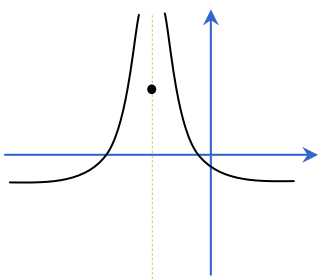

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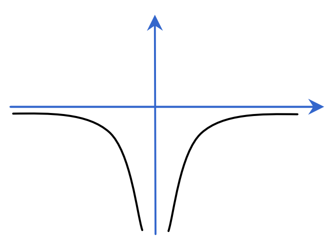

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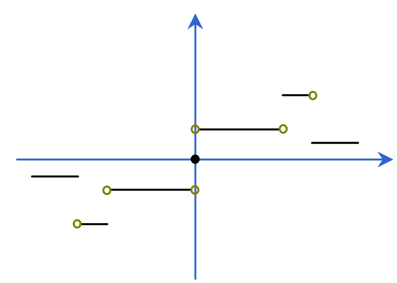

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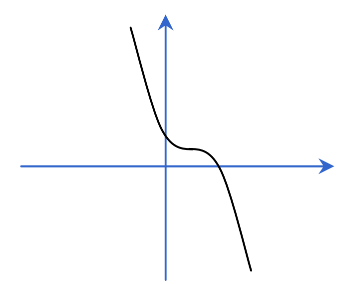
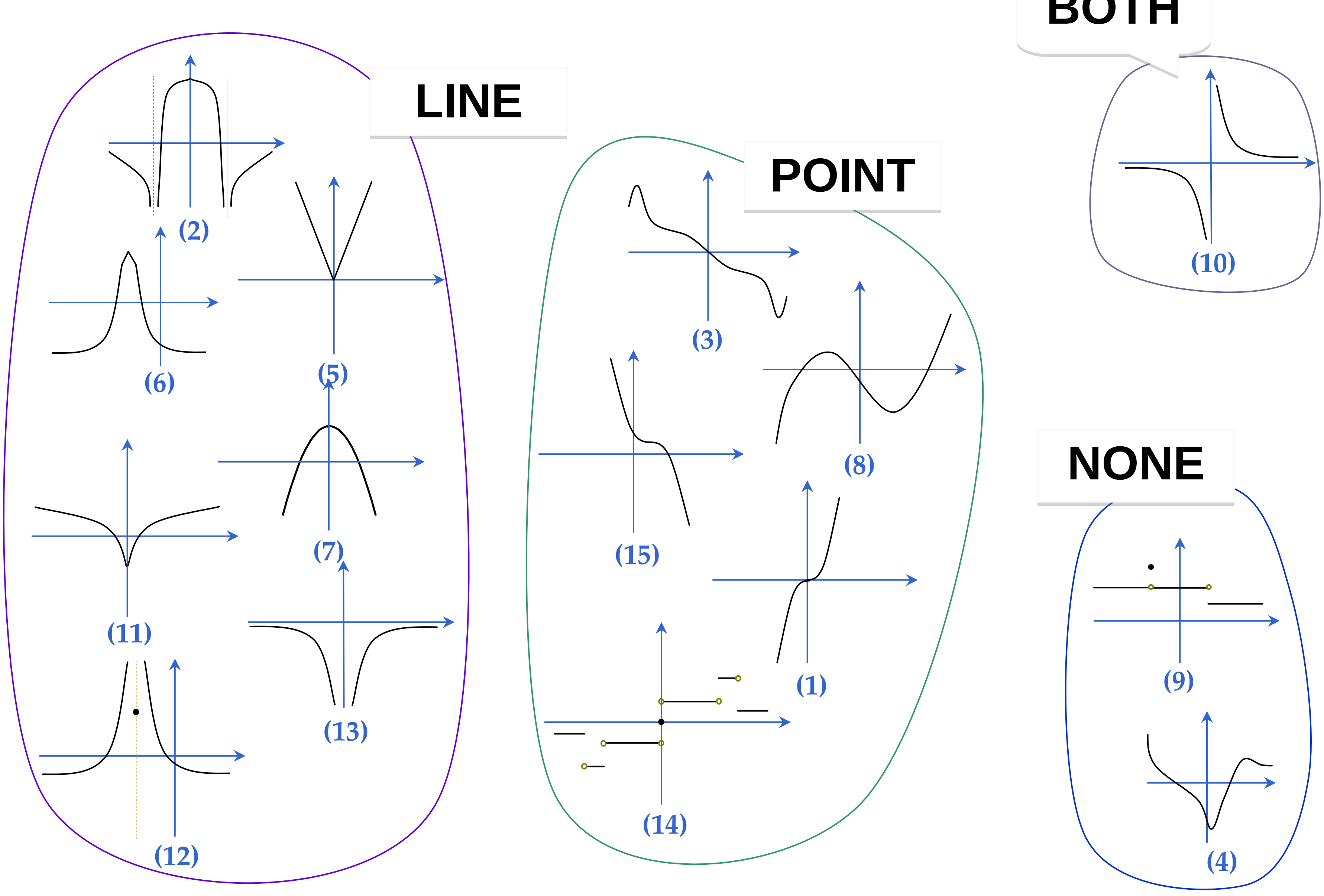


(15)

Orit Zaslavsky, 2008

Symmetry




e

Creating problems, exploring structure

Students:

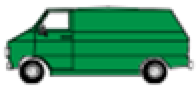
- Explore the effect of varying the constraints in a problem.
- Explore how one variable depends upon another.
- Devise variations of problems for other students to solve.
- Are creative and ‘own’ problems.
- Take on the role of teacher and explainer.
- Exemplify the ‘doing’ and ‘undoing’ processes of mathematics.

“Standard” question

Van hire

Sanjay wants to hire a van to move some furniture.
He obtains the following information from two hire companies.

Bujit’s Van Hire



£30 for the first 50 miles

Every mile after that costs an extra 20p

Hurt’s vans

You only pay for the miles you travel.

Miles travelled	50	100	150	200
Hire charge	£16	£32	£48	£64

1. How much do Hurt’s vans cost per mile?
2. Sanjay expects to travel 175 miles.
   Which company has the lower charge for this distance?
   You must show all your working.

An open template for a new question

Cath wants to hire a car for a weekend.

She obtains the following information from two hire companies.

.... Car Hire

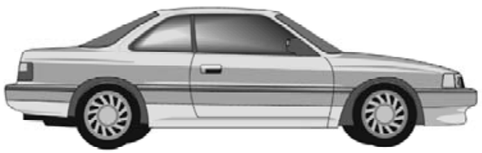
£ ....for the first

..............miles.

Every mile after that costs an

extra .... p.

.... Car Hire



Miles travelled				
Hire charge				

. . . .

. . . .

. . . .

Doing and undoing processes

Doing: **The problem poser…**	**Undoing:** **The problem solver…**
generates an equation step-by-step, ‘doing the same to both sides’.	solves the resulting equation.
draws a rectangle and calculates its area and perimeter.	tries to draw a rectangle with the given area and perimeter.
writes down an equation of the form $y=mx+c$ and plots a graph.	tries to find an equation that fits the resulting graph.

Doing and undoing processes

Doing: **The problem poser…**	**Undoing:** **The problem solver…**
expands an algebraic expression such as (x+3)(x-2).	factorises the resulting expression: x^2+x-6.
writes down a polynomial and differentiates it.	integrates the resulting function.
writes down five numbers and finds their mean, median and range.	tries to find five numbers with the given mean, median and range.

French Class

A teacher runs a French evening class for 20 weeks.

The class meets each week in the village hall.

It costs the teacher £ $\overset{h}{\boxed{200}}$ to hire the hall for the whole course.

The class contains $\overset{n}{\boxed{10}}$ students.

Each student pays the teacher a single fee of £ $\overset{f}{\boxed{60}}$ for the course.

The teacher makes £ $\overset{p}{\boxed{400}}$ profit at the end of the course.

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How can we calculate the profit?

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How can we calculate the profit?

$$p = 10 \times 60 - 200$$

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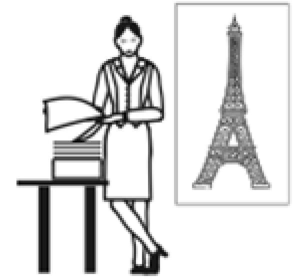
How can we calculate the amount students are charged?

French Class



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How can we calculate the amount students are charged?

$$f = \frac{400 + 200}{10}$$

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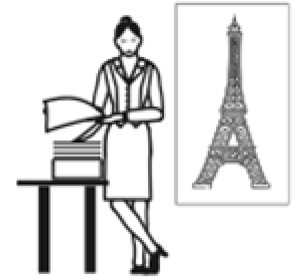
How does the profit depend on the number of students?

French Class



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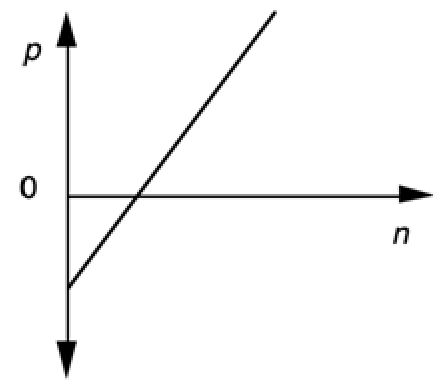
The teacher makes £ $\overset{p}{\boxed{}}$ profit at the end of the course.

n	0	2	4	6	8	10
p	-200	-80	40	160	280	400



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The class contains n students.

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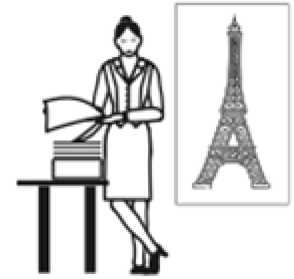
The teacher makes £ p profit at the end of the course.

What general relationships can you write down ?

French Class



A teacher runs a French evening class for 20 weeks.

The class meets each week in the village hall.

It costs the teacher £ h to hire the hall for the whole course.

The class contains n students.

Each student pays the teacher a single fee of £ f for the course.

The teacher makes £ p profit at the end of the course.

$$p = nf - h \qquad n = \frac{p+h}{f} \qquad f = \frac{p+h}{n} \qquad h = nf - p$$

What has been the impact?

Evidence of impact

- Careful research and observation of teachers and classrooms - academic papers.
- Take-up
- Inspection reports

Adopting the principles

- Use cooperative small group work.
- Use rich collaborative tasks.
- Use higher-order questions.
- Build on knowledge students already have.
- Create connections between topics.
- Emphasise reasons rather than answers.
- Expose and discuss misconceptions.
- Use technology in appropriate ways.

Algebra study with 45 teachers

When discussion activities were used:

- Teachers reported changes to their beliefs about teaching and learning.
- Student algebra learning increased with greater use of the activities and with more student-centred implementations.
- Significant (but small) improvement in self-efficacy of students.

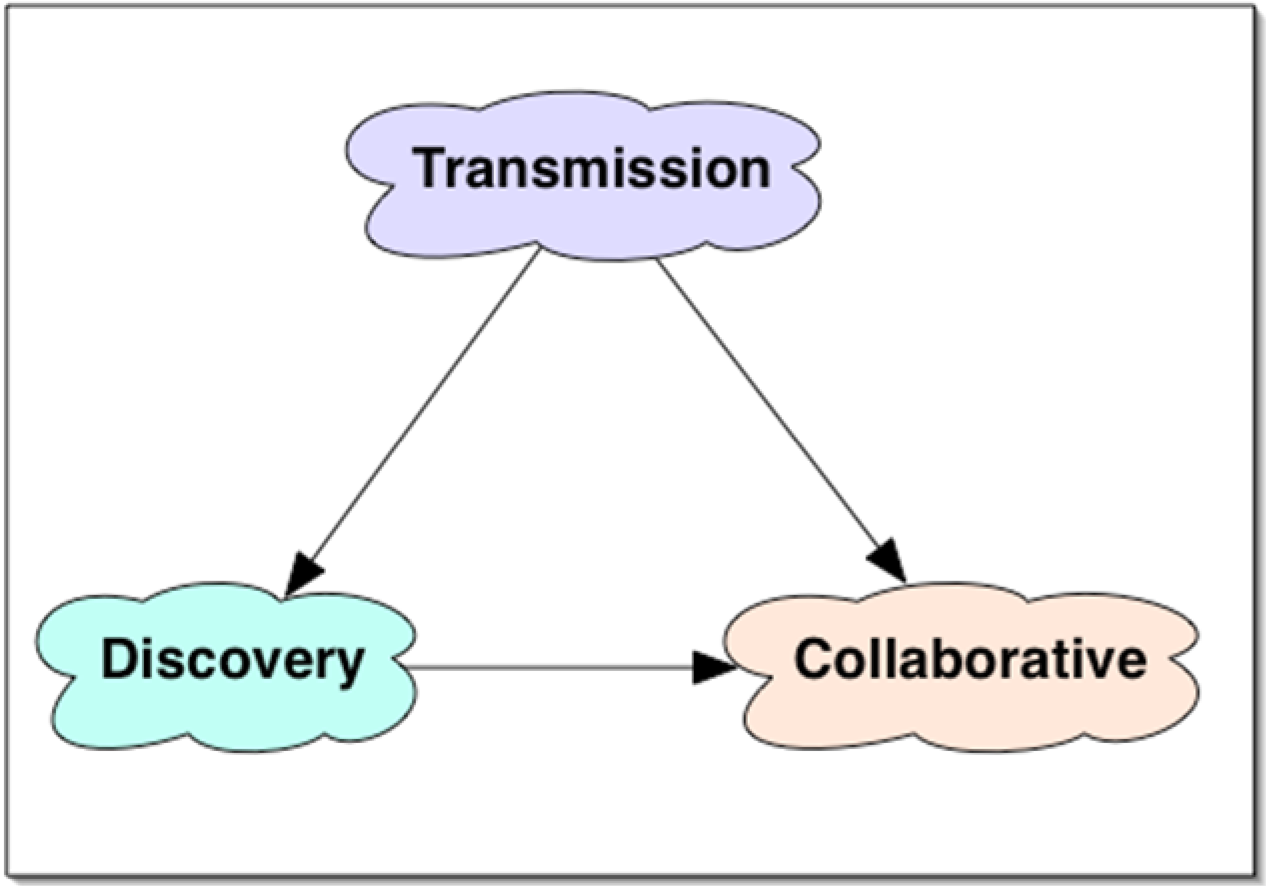
When discussion activities were *not* used:

- Significant regression in confidence and effectance motivation,
- Increased algebra anxiety, passive learning behaviours.

More or less effective?

More effective	**Less effective**
Offer challenge before help	Offer help before challenge
Discuss ways of working	Tell them what to do
Elicit interpretations and methods	Elicit facts and answers
Pause after questions and answers	Give answers if none forthcoming
Listen befoe intervening	Intervene before listening
When students cannot explain, dont let them off the hook!	When students cannot explain, explain for them.
Leave some discussions unresolved	Feel need to resolve every discussion

Evolution of beliefs



Transmission > transmission

"I feel that these (tasks) are very good for learning breakthroughs, but I don’t think they are going to get the bulk of my students through their exams. I think you need a ‘crammed’ approach. This is the big issue for me. I’d be quite happy to use these materials every lesson, the time went ‘like that’ and it’s great to see people not yawning and actually enjoying themselves. You don’t have discipline problems.”

“**But, I feel that for an exam, I’ve got to feel that I am giving them the knowledge that they need to pass that exam and I feel that I can do that through the traditional approaches and a bit of bullying.**"

Discovery > collaborative

"The thing that I liked is that they got confused. It is something they can work on. If they had put all their statements in the right place, there is no lesson there. I'm very glad they did get confused, because then they started to think about it. Some of the groups were then starting to discuss 'what does that mean?'"

Transmission > Collaborative

"A lot of misconceptions came out. I was surprised by how well the discussion went. This class was inspected and this was one lesson he (the inspector) was impressed by."

Inspection reports

These materials encouraged teachers to be more reflective and offered strategies to encourage students to think more independently. They encouraged discussion and active learning in AS, A level and GCSE lessons.

While some colleges were just dipping into the resources, a few had used the full package to transform teaching and learning across an entire mathematics team.

(OfSTED 2006)

Learners enjoy challenge

- *You can’t just do it, you have to work it out, which is good. Better it being that way or you won’t learn nowt.*
- *This challenges what you know and makes you think about what you are doing.*
- *I don’t like going below what I can do. I love challenges. I want to challenge my strengths and weaknesses, I want to just challenge everything.*
- *If you have just learned it and someone asks you for help and you can explain it then you know you have learned it.*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Support for leaders and managers
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IT
SHD
Recent releases
Engineering
Construction
IT
SHD
Creative and media
Customer care
Enterprise
Expert learner
Health and safety
Learning Maths

Thank you !

Research book and algebra materials from:

http://www.mathshell.com

Collaborative learning materials may be downloaded from:

http://teachingandlearning.qia.org.uk/#math

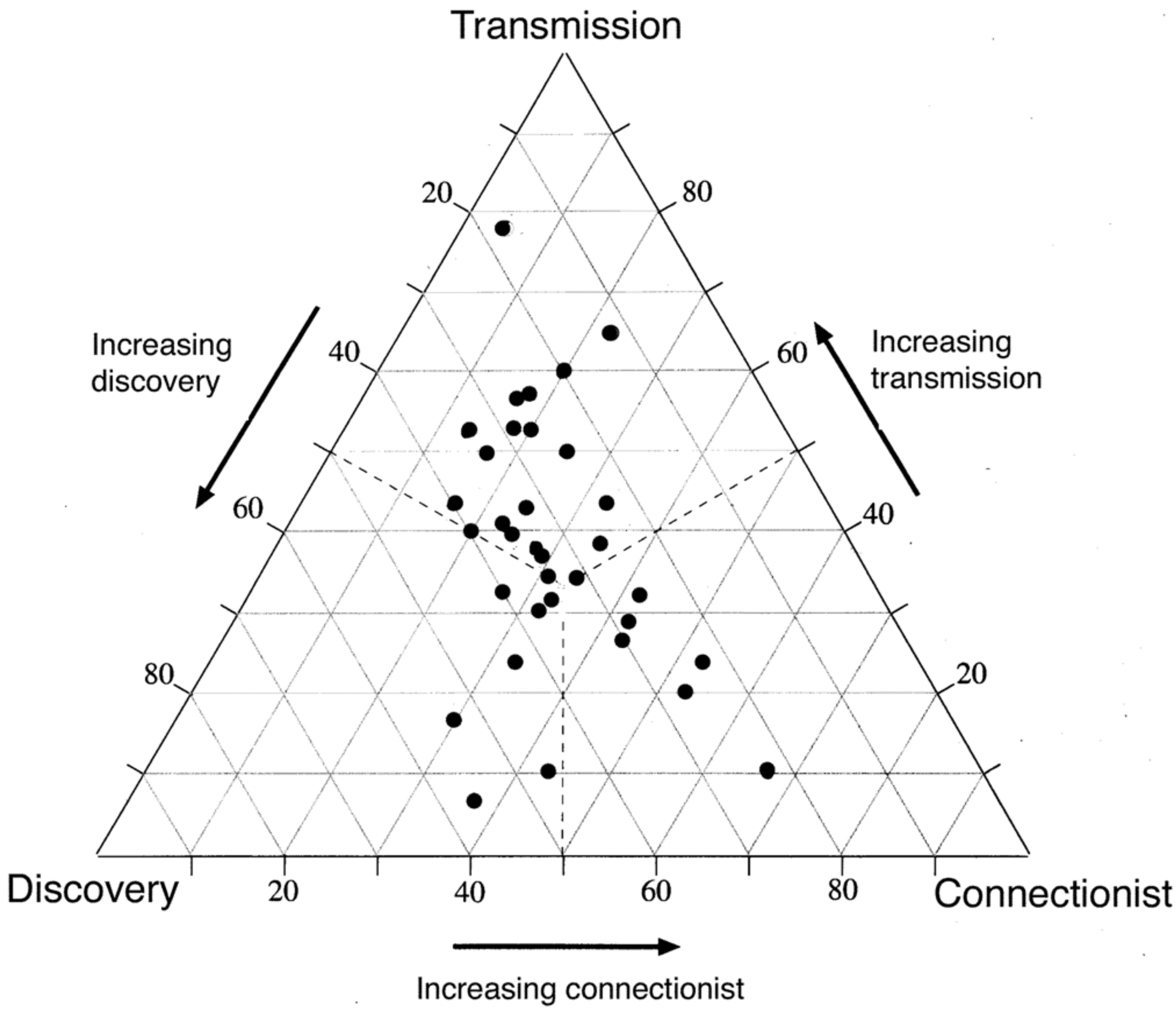
Ofsted inspection (May 2006)

- May 2006: Ofsted visited 26 schools, sixth form colleges and general further education colleges
- To determine factors leading to high achievement, motivation and participation in 14–19 mathematics, and factors which act against high achievement.

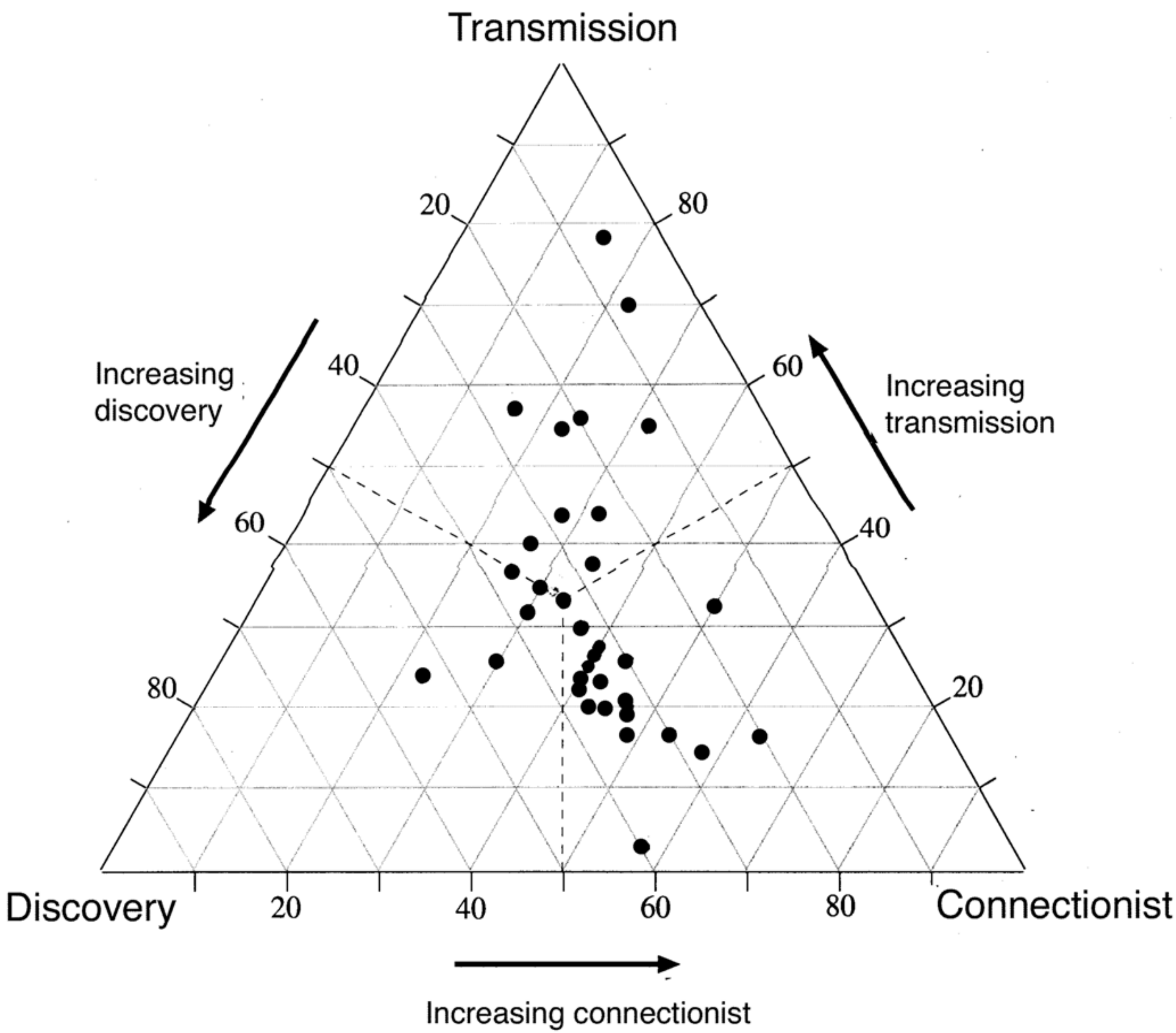
‘Discovery’ belief system

- **Mathematics** is seen as
  - a creative subject in which the teacher should take a facilitating role, allowing students to create their own concepts and methods.
- **Learning** is seen as:
  - an individual activity based on practical exploration and reflection
- **Teaching** is seen as
  - assessing when a student is ready to learn;
  - providing a stimulating environment to facilitate exploration;
  - avoiding misunderstandings by the careful sequencing of experiences.

Professed beliefs of teachers (pre)



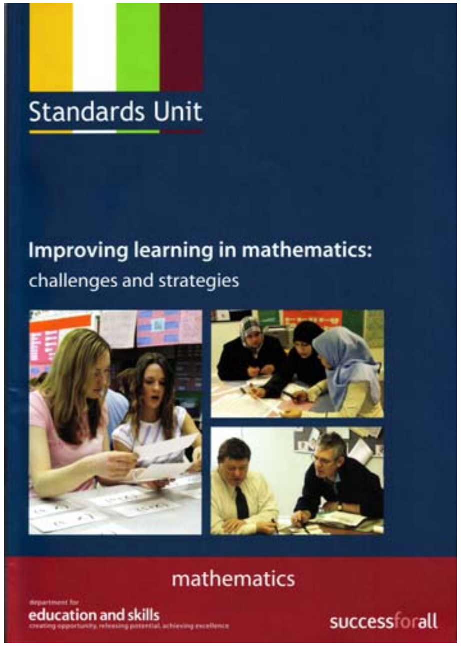
Professed beliefs of teachers (post)



What did the design of tasks contribute?

- fostered sustained collaborative work;
- encouraged teachers to challenge students
- confronted students with specific, conceptual obstacles;
- provided teachers with task 'genres' that embody mathematical thinking processes:
- influenced the nature of questions that teachers and students ask of each other
- moved the classroom agenda away from 'completion' towards 'comprehension'.

- Third design
  Improving Learning in Mathematics
  - Expansion of resources and new cohort
    (both GCSE and A level)
    (2 years - 200 teachers, 40 organisations)
  - 24000 copies of 'box' made.
  - Sent by invitation to all post 16 providers (incl. prisons etc)
  - Now available to all schools



- Fourth design
  Thinking through Mathematics
  - **For adult learners**
    (1 year - 24 teachers)